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Structural displacement function in spherical coordinates

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Governing equations (from S.P.Timoshenko, "Theory of Elasticity" and PeiChiChou & N.J.Pagano, "Elasticity, tensor, dyadic, and Engineering approaches")

$with(Physics[Vectors]):$

$Cte := \left[G = \frac{E}{2(1+\nu)}, \lambda = \frac{2\,G\cdot\nu}{(1-2\,\nu)}\right]; assume(E > 0) : assume(0 \geq \nu < 0.5) :$

$$\left[G = \frac{1}{2}\,\frac{E}{1+\nu}, \lambda = \frac{2\,G\,\nu}{1-2\,\nu}\right] \tag{1.1}$$

$F := \rho\frac{4\,\pi\,r^3}{3}\,\frac{\partial^2}{\partial\,t^2}\,u(t, r, \phi, \theta); assume(m > 0) :$

$$\frac{4}{3}\,\rho\,\pi\,r^3\left(\frac{\partial^2}{\partial t^2}\,u(t, r, \phi, \theta)\right) \tag{1.2}$$

$NS1 := G\,\nabla^2\boldsymbol{u}(t, r, \phi, \theta) + (\lambda + G)\,\nabla\nabla\cdot\boldsymbol{u}(t, r, \phi, \theta) + \boldsymbol{F} = 0$

$$\frac{1}{r^2\sin(\theta)^2}\Bigg(G\Bigg(r^2\sin(\theta)^2\left(\frac{\partial^2}{\partial r^2}\,u(t, r, \phi, \theta)\right) + 2\,r\sin(\theta)^2\left(\frac{\partial}{\partial r}\,u(t, r, \phi, \theta)\right) + \left(\frac{\partial^2}{\partial \theta^2}\,u(t, r, \phi, \theta)\right)\sin(\theta)^2 + \sin(\theta)\left(\frac{\partial}{\partial \theta}\,u(t, r, \phi, \theta)\right)\cos(\theta) + \frac{\partial^2}{\partial \phi^2}\,u(t, r, \phi, \theta)\Bigg)\Bigg) + \frac{1}{r^2\sin(\theta)^2}\Bigg((\lambda + G)\Bigg(r^2\sin(\theta)^2\left(\frac{\partial^2}{\partial r^2}\,u(t, r, \phi, \theta)\right) + 2\,r\sin(\theta)^2\left(\frac{\partial}{\partial r}\,u(t, r, \phi, \theta)\right) + \left(\frac{\partial^2}{\partial \theta^2}\,u(t, r, \phi, \theta)\right)\sin(\theta)^2 + \sin(\theta)\left(\frac{\partial}{\partial \theta}\,u(t, r, \phi, \theta)\right)\cos(\theta) + \frac{\partial^2}{\partial \phi^2}\,u(t, r, \phi, \theta)\Bigg)\Bigg) + \frac{4}{3}\,\rho\,\pi\,r^3\left(\frac{\partial^2}{\partial t^2}\,u(t, r, \phi, \theta)\right) = 0 \tag{1.3}$$

$\left(collect\left(NS1, \left[r, r^2, \frac{\partial}{\partial r}\,u(t, r, \phi, \theta), \frac{\partial^2}{\partial r^2}\,u(t, r, \phi, \theta)\right]\right)\right)$

$$\frac{4}{3}\,\rho\,\pi\,r^3\left(\frac{\partial^2}{\partial t^2}\,u(t, r, \phi, \theta)\right) + (2\,G + \lambda)\left(\frac{\partial^2}{\partial r^2}\,u(t, r, \phi, \theta)\right) + \frac{(4\,G + 2\,\lambda)\left(\frac{\partial}{\partial r}\,u(t, r, \phi, \theta)\right)}{r} + \frac{1}{r^2}\left(\frac{1}{\sin(\theta)^2}\left(G\left(\frac{\partial^2}{\partial \phi^2}\,u(t, r, \phi, \theta) + \left(\frac{\partial^2}{\partial \theta^2}\,u(t, r, \phi, \theta)\right)\sin(\theta)^2 + \sin(\theta)\left(\frac{\partial}{\partial \theta}\,u(t, r, \phi, \theta)\right)\cos(\theta)\right)\right)\right) \tag{1.4}$$

$$+\frac{1}{\sin(\theta)^2}\left((\lambda+G)\left(\frac{\partial^2}{\partial\phi^2}u(t,r,\phi,\theta)+\left(\frac{\partial^2}{\partial\theta^2}u(t,r,\phi,\theta)\right)\sin(\theta)^2\right.\right.$$

$$\left.\left.+\sin(\theta)\left(\frac{\partial}{\partial\theta}u(t,r,\phi,\theta)\right)\cos(\theta)\right)\right)\right)=0$$

$$\left(collect\left(NS1,\left[r,r^2,r^3,\frac{\partial}{\partial r}u(t,r,\phi,\theta),\frac{\partial^2}{\partial r^2}u(t,r,\phi,\theta),\frac{\partial^2}{\partial\phi^2}u(t,r,\phi,\theta),\frac{\partial^2}{\partial\theta^2}u(t,r,\phi,\theta)\right]\right)\right)$$

$$\frac{4}{3}\rho\pi r^3\left(\frac{\partial^2}{\partial t^2}u(t,r,\phi,\theta)\right)+(2G+\lambda)\left(\frac{\partial^2}{\partial r^2}u(t,r,\phi,\theta)\right)$$

$$+\frac{(4G+2\lambda)\left(\frac{\partial}{\partial r}u(t,r,\phi,\theta)\right)}{r}+\frac{1}{r^2}\left(\left(\frac{G}{\sin(\theta)^2}+\frac{\lambda+G}{\sin(\theta)^2}\right)\left(\frac{\partial^2}{\partial\phi^2}u(t,r,\phi,\theta)\right)+(2G+\lambda)\left(\frac{\partial^2}{\partial\theta^2}u(t,r,\phi,\theta)\right)+\frac{G\left(\frac{\partial}{\partial\theta}u(t,r,\phi,\theta)\right)\cos(\theta)}{\sin(\theta)}\right.$$

$$\left.+\frac{(\lambda+G)\left(\frac{\partial}{\partial\theta}u(t,r,\phi,\theta)\right)\cos(\theta)}{\sin(\theta)}\right)=0 \quad \textbf{(1.5)}$$

$$NS1r:=\frac{4}{3}\rho\pi r^3\left(\frac{\partial^2}{\partial t^2}u(t,r)\right)+(2G+\lambda)\left(\left(\frac{\partial^2}{\partial r^2}u(t,r)\right)+\frac{2}{r}\left(\frac{\partial}{\partial r}u(t,r)\right)\right)$$

$$\frac{4}{3}\rho\pi r^3\left(\frac{\partial^2}{\partial t^2}u(t,r)\right)+(2G+\lambda)\left(\frac{\partial^2}{\partial r^2}u(t,r)+\frac{2\left(\frac{\partial}{\partial r}u(t,r)\right)}{r}\right) \quad \textbf{(1.6)}$$

$$NSIpt:=\frac{1}{r^2\sin(\theta)^2}\left((\lambda+2G)\left(\left(\frac{\partial^2}{\partial\phi^2}u(t,\phi,\theta)\right)+\left(\frac{\partial^2}{\partial\theta^2}u(t,\phi,\theta)\right)\sin(\theta)^2+\sin(\theta)\left(\frac{\partial}{\partial\theta}u(t,\phi,\theta)\right)\cos(\theta)\right)\right)$$

$$\frac{1}{r^2\sin(\theta)^2}\left((2G+\lambda)\left(\frac{\partial^2}{\partial\phi^2}u(t,\phi,\theta)+\left(\frac{\partial^2}{\partial\theta^2}u(t,\phi,\theta)\right)\sin(\theta)^2+\sin(\theta)\left(\frac{\partial}{\partial\theta}u(t,\phi,\theta)\right)\cos(\theta)\right)\right) \quad \textbf{(1.7)}$$