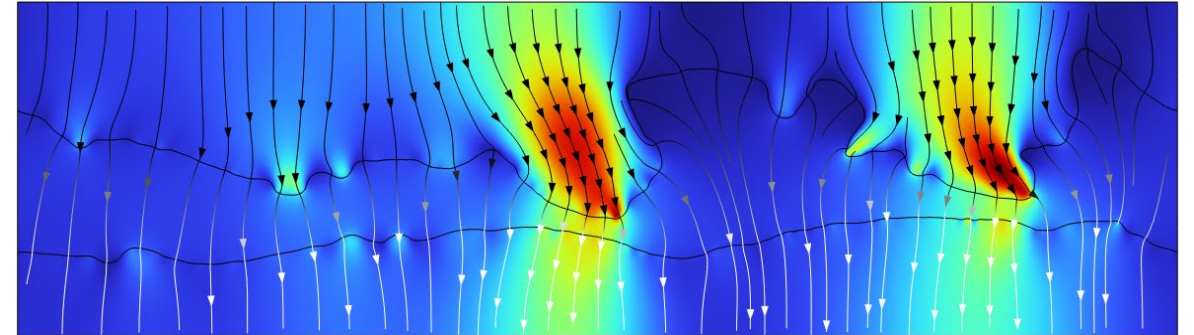
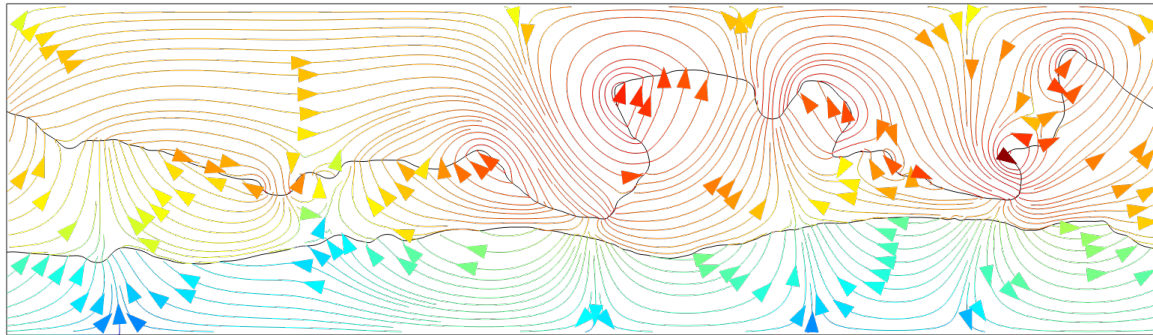


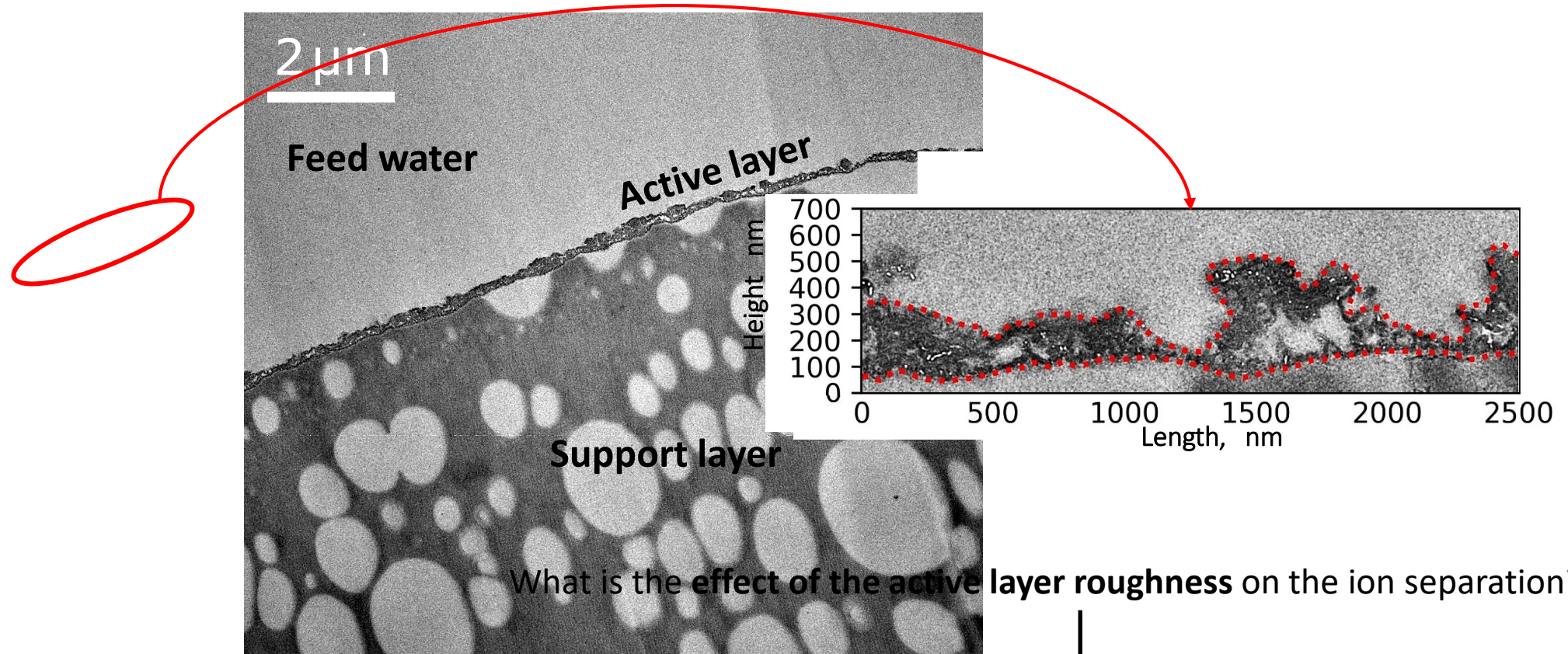
# 2-D ion transport modelling of water desalination by reverse osmosis (RO) considering the real membrane effect

**Fernan Martinez**

Bastiaan Blankert Valentina-Elena Musteață  
Cristian Picioreanu



# Describing the problem

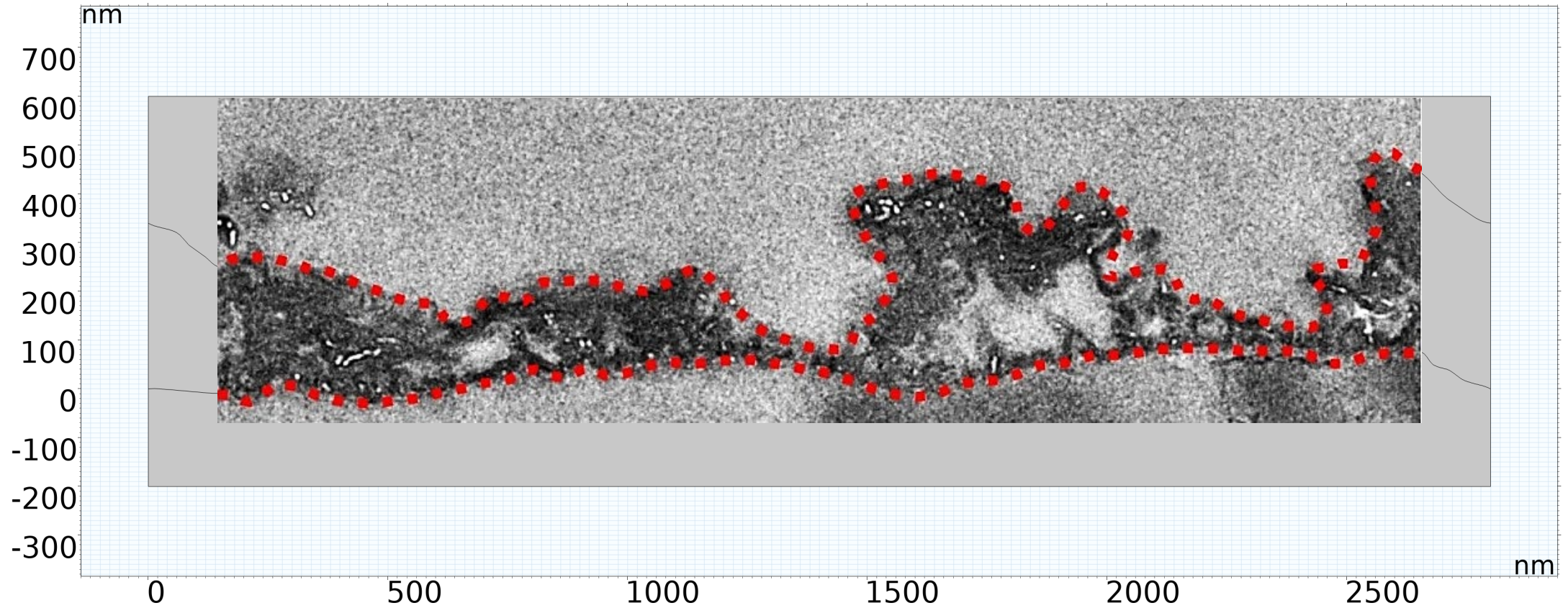


What is the effect of the active layer roughness on the ion separation?

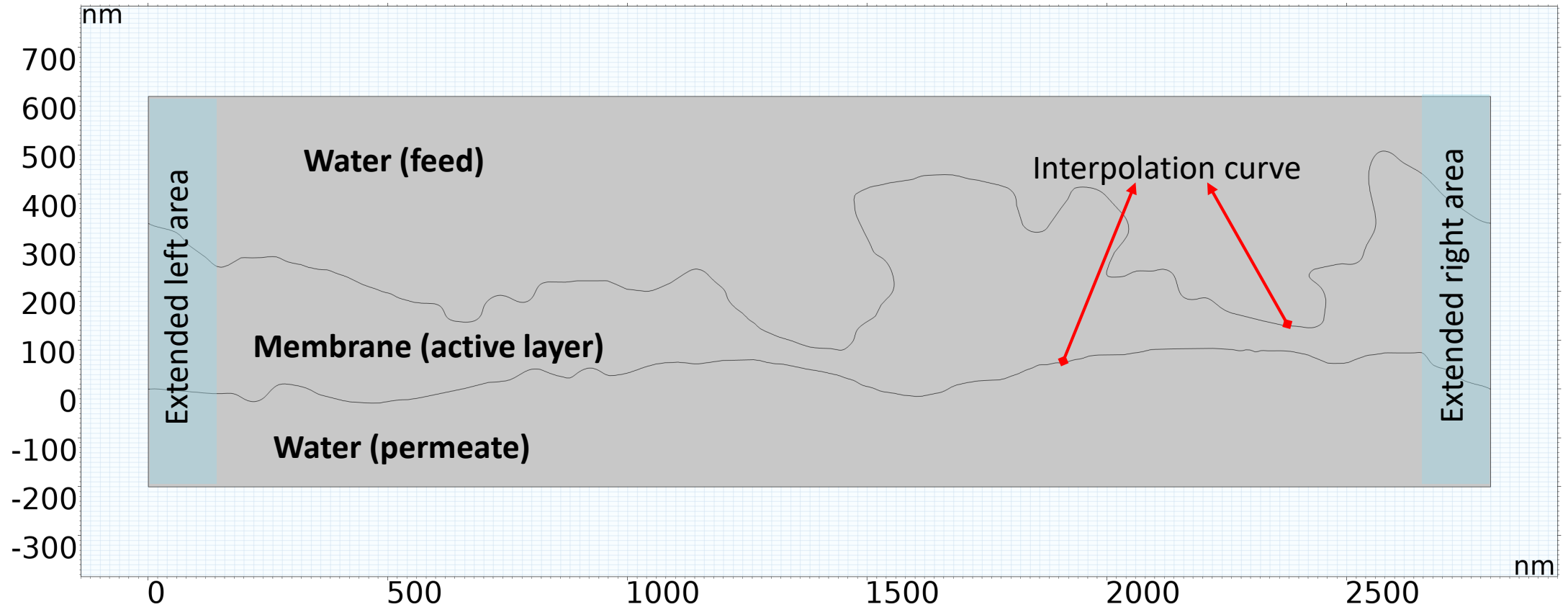


Compute  $\text{Na}^+$  and  $\text{Cl}^-$  transport through the active layer with real 2-D geometry

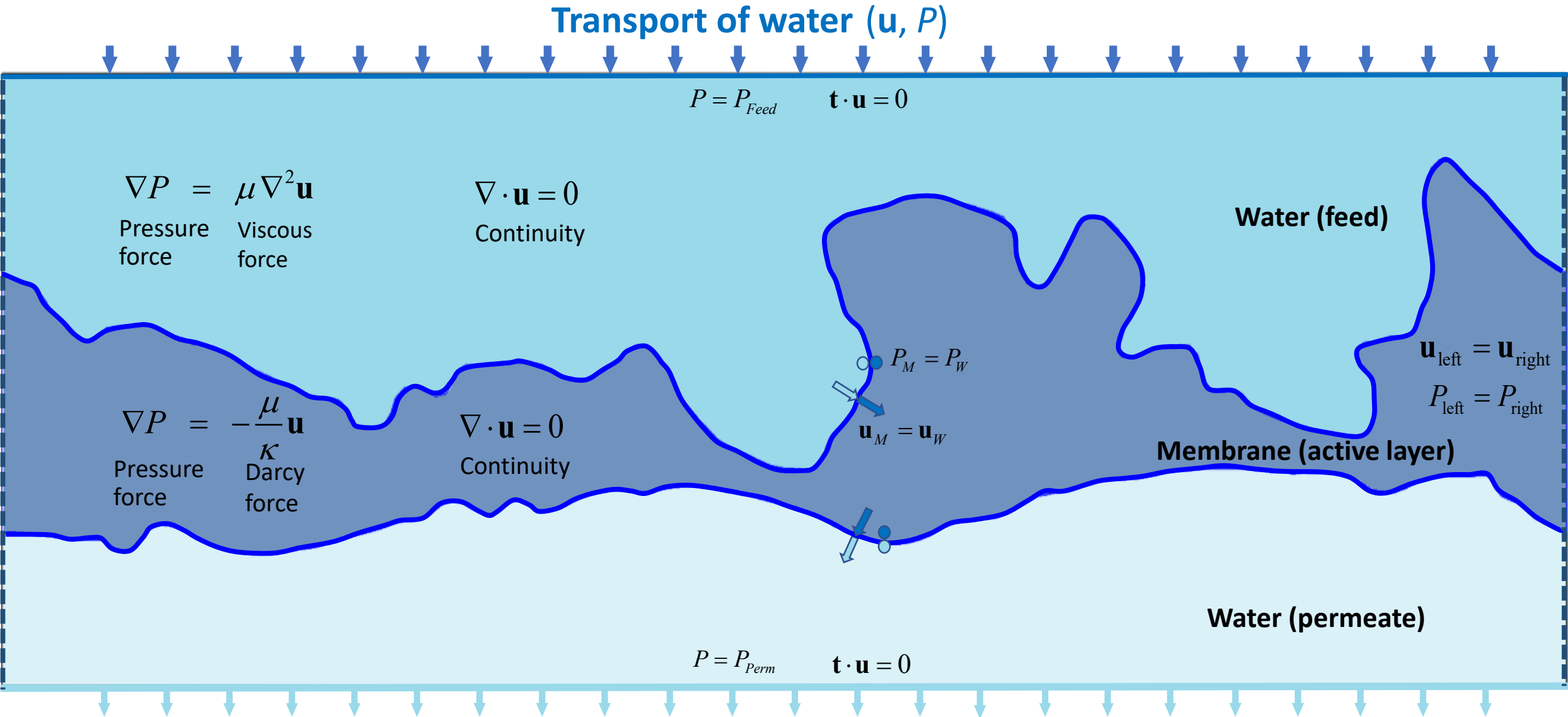
# Geometry




# Geometry



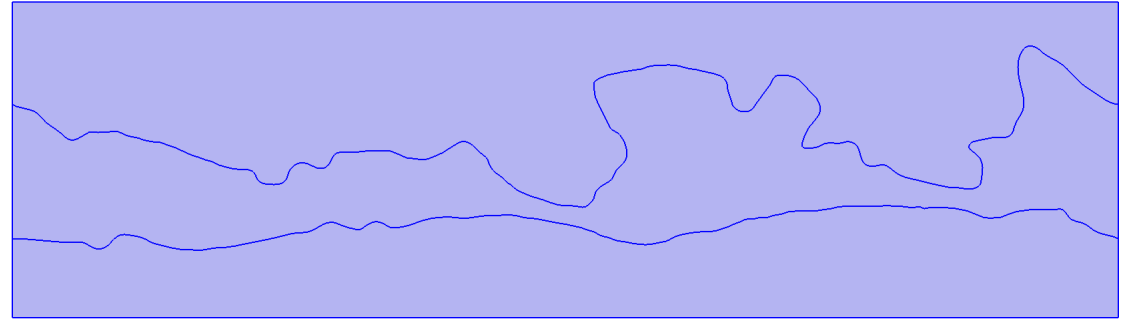
# Model description. Transport of water: creeping flow



# Model implementation. Transport of water: creeping flow

- ▲  Creeping Flow (*spf*)
  - Feed and permeate properties
  - Initial Values 1
  - Wall 1
- ▲  Porous Medium
  - Fluid
  - Porous Membrane
- Inlet pressure feed
- Outlet pressure permeate
- Periodic Flow Condition

Domain selection: all domains



▼ Dependent Variables

Velocity field:

u

Velocity field components:

u


v

~~w~~

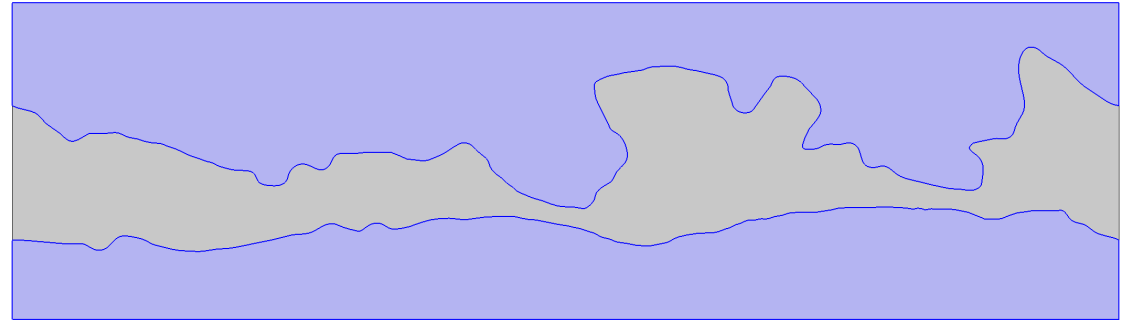
Pressure:

p

# Model implementation. Transport of water: creeping flow

- ▲  Creeping Flow (*spf*)
  - Feed and permeate properties
  - Initial Values 1
  - Wall 1
- ▲  Porous Medium
  - Fluid
  - Porous Membrane
- Inlet pressure feed
- Outlet pressure permeate
- Periodic Flow Condition

## Domain selection: feed and permeate domain



▼ Fluid Properties

Density:

$\rho$   ▼

kg/m<sup>3</sup>


— Constitutive relation —

▼

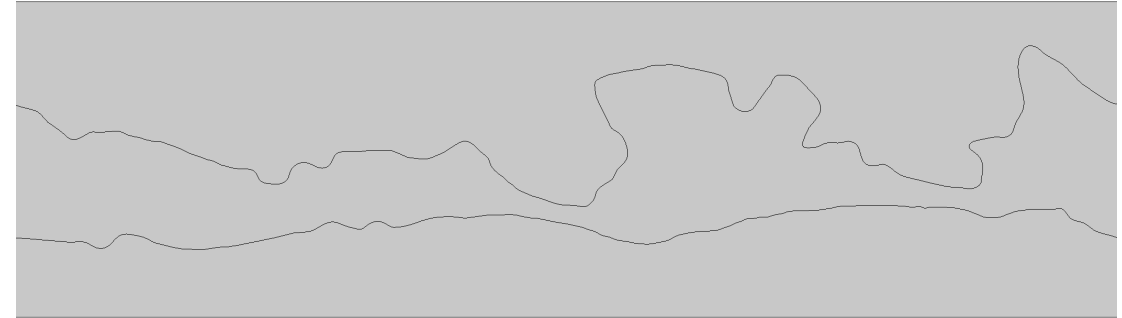
$\mu$   ▼

Pa·s

# Model implementation. Transport of water: creeping flow











- ▲  Creeping Flow (*spf*)
    - Feed and permeate properties
    - Initial Values 1
    - Wall 1
  - ▲  Porous Medium
    - Fluid
    - Porous Membrane
  - Inlet pressure feed
  - Outlet pressure permeate
  - Periodic Flow Condition
- Overridden by

**Boundary selection:** all boundaries were overridden

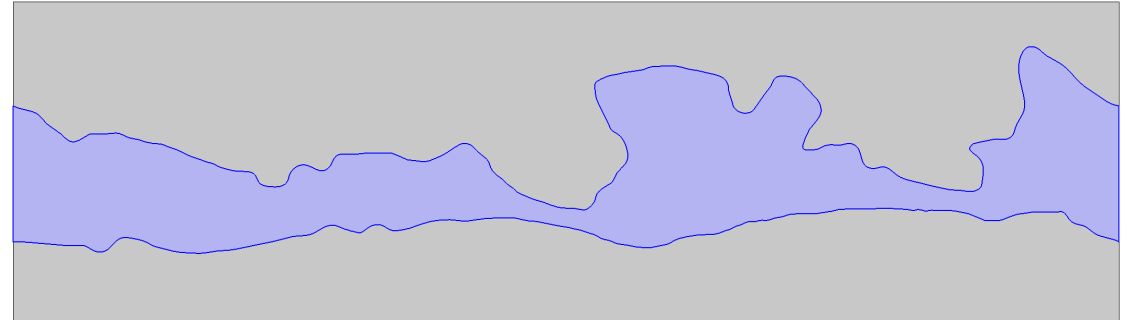




# Model implementation. Transport of water: creeping flow

- ▲  Creeping Flow (*spf*)
  -  Feed and permeate properties
  -  Initial Values 1
  -  Wall 1
- ▲  Porous Medium
  -  Fluid
  -  Porous Membrane
-  Inlet pressure feed
-  Outlet pressure permeate
-  Periodic Flow Condition

## Domain selection: membrane



▼ Matrix Properties

Porosity:

$\epsilon_p$   ▼

Permeability model:











▼

Permeability:

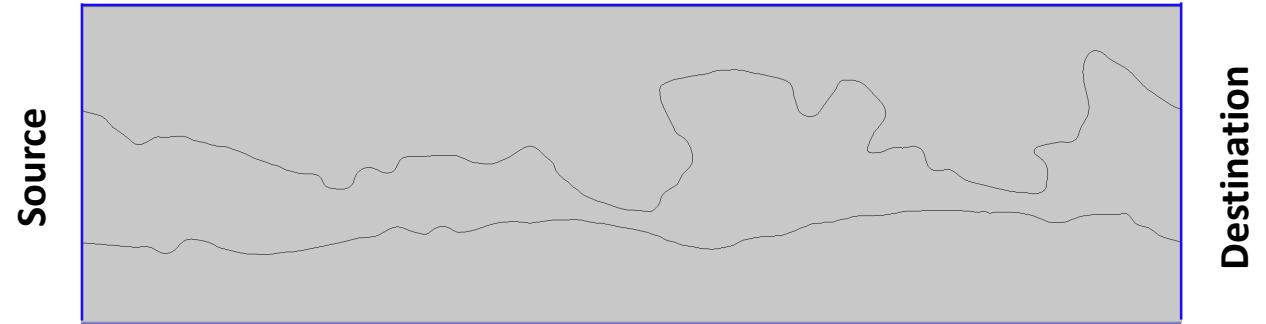
$\kappa$   ▼

▼

# Model implementation. Transport of water: creeping flow

- ▲  Creeping Flow (*spf*)
  -  Feed and permeate properties
  -  Initial Values 1
  -  Wall 1
- ▲  Porous Medium
  -  Fluid
  -  Porous Membrane
-  Inlet pressure feed
-  Outlet pressure permeate
-  Periodic Flow Condition

## Boundaries selection: external boundaries



▼ Boundary Condition

Pressure

▼ Pressure Conditions

Pressure:

Static

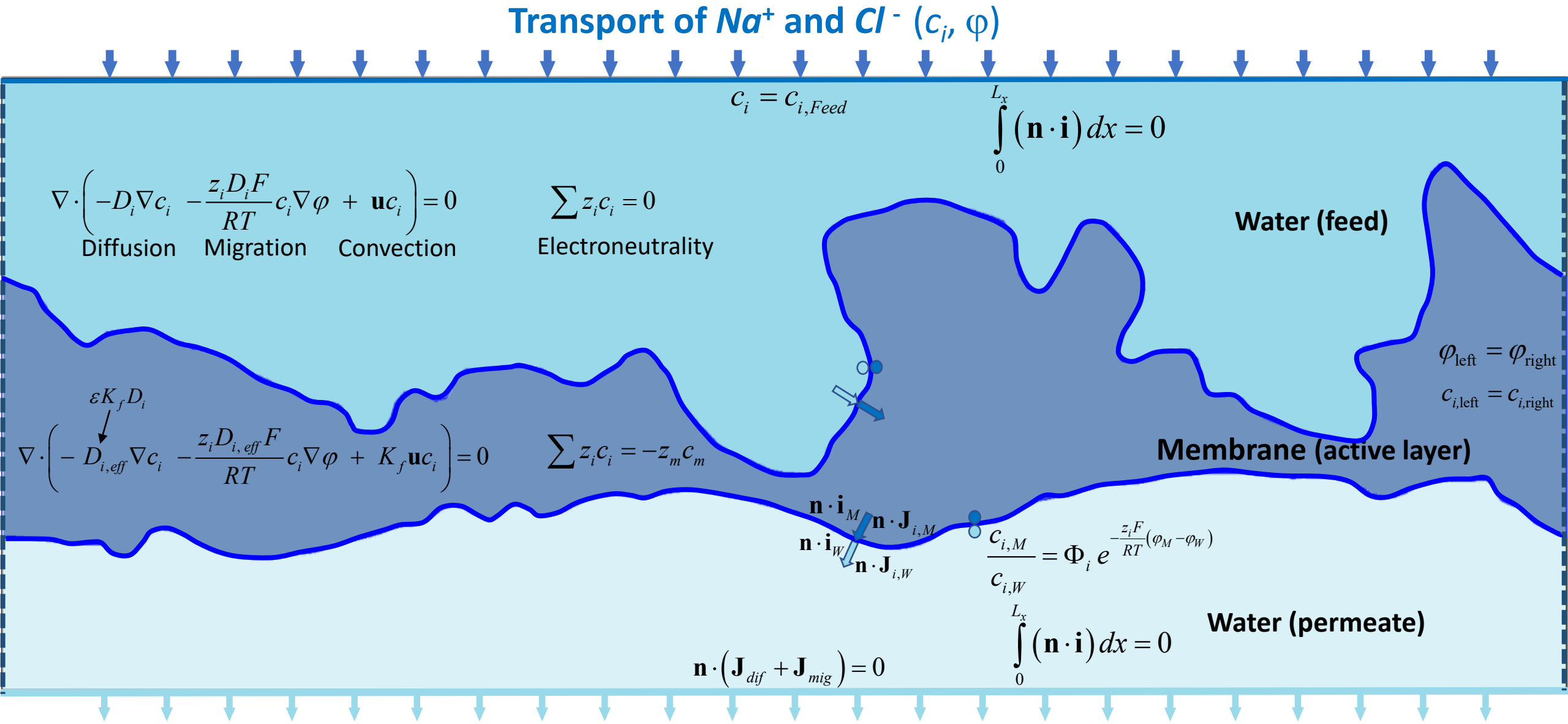
$p_0$   Pa

Suppress backflow

Flow direction:

Normal flow

# Model description. Transport of species: tertiary current distribution, Nernst-Plank

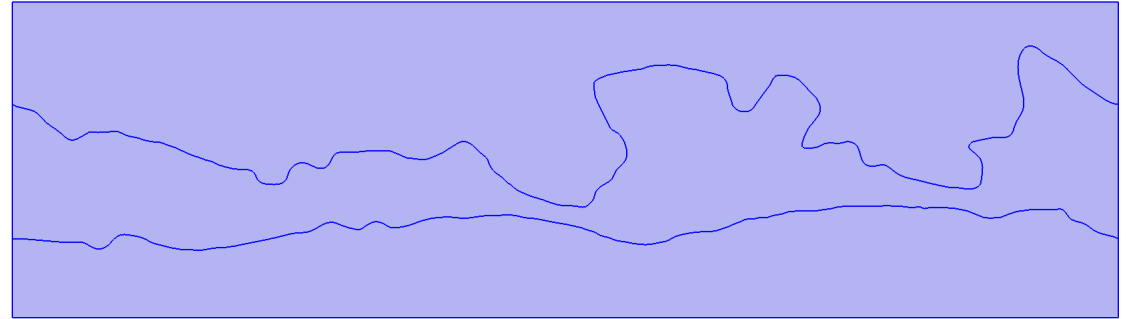


# Model implementation. Transport of species: tertiary current distribution, Nernst-Planck

## ▲ Tertiary Current Distribution, Nernst-Planck (*tcd*)

- Species Charges
- ▶  Electrolyte Liquid (Feed and Permeate)
  - No Flux
  - Insulation
  - Initial Values Membrane
  - Initial Values Feed
  - Initial Values Permeate
- ▲  Ion Exchange Membrane
  - $\frac{\partial u}{\partial t} = f$  Equation View
  - Concentration feed
  - Only convection on permeate
  - Periodic Condition
  - Electrolyte Current Feed
  - Electrolyte Current Permeate
  - Electrolyte Potential in one point

## Domain selection: all domains



### ▼ Dependent Variables

|                        |       |
|------------------------|-------|
| Number of species:     | 2     |
| Concentrations:        | c_cl  |
|                        | c_na  |
|                        |       |
|                        | + ☰   |
| Electrolyte potential: | V     |
| Electric potential:    | dummy |

# Transport of species: tertiary current distribution, Nernst-Planck

## ▲ Tertiary Current Distribution, Nernst-Planck (*tcd*)

Species Charges

▶  Electrolyte Liquid (Feed and Permeate)

No Flux

Insulation

Initial Values Membrane

Initial Values Feed

Initial Values Permeate

▲  Ion Exchange Membrane

$\frac{\partial u}{\partial t} = f$  Equation View

Concentration feed

Only convection on permeate

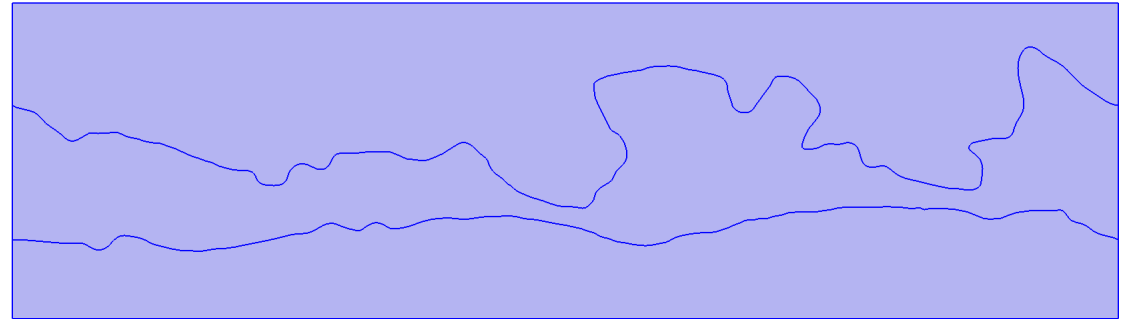
Periodic Condition

Electrolyte Current Feed

Electrolyte Current Permeate

Electrolyte Potential in one point

Domain selection: all domains



▼ Charge

$z_{c\_cl}$   1

$z_{c\_na}$   1

# Transport of species: tertiary current distribution, Nernst-Planck

## ▾ Tertiary Current Distribution, Nernst-Planck (*tcd*)

Species Charges

### ▸ Electrolyte Liquid (Feed and Permeate)

No Flux


Insulation

Initial Values Membrane

Initial Values Feed

Initial Values Permeate

### ▾ Ion Exchange Membrane

 Equation View

Concentration feed

Only convection on permeate

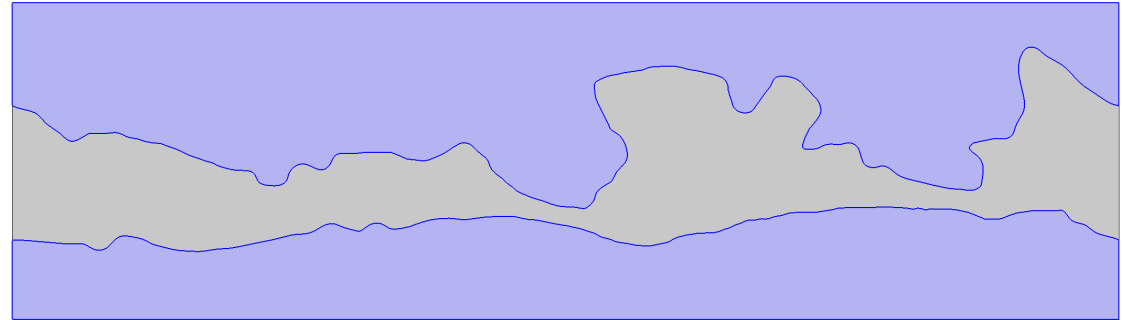
Periodic Condition

Electrolyte Current Feed

Electrolyte Current Permeate

Electrolyte Potential in one point

## Domain selection: feed and permeate



### ▾ Diffusion

Material:

None

Diffusion coefficient:

$D_{c\_cl}$  User defined

$D_{cl} = 2.032E-9$  m<sup>2</sup>/s

Isotropic

Diffusion coefficient:

$D_{c\_na}$  User defined

$D_{na} = 1.334E-9$  m<sup>2</sup>/s

Isotropic

### ▾ Migration in Electric Field

Mobility:

Nernst-Einstein relation



$$u_{m,j} = \frac{D_j}{RT}$$

### ▾ Convection

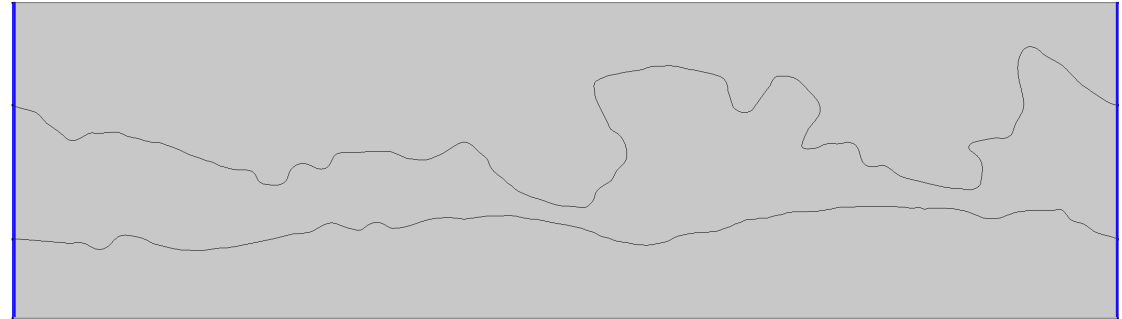
Velocity field:

$u$  Velocity field (spf)

# Transport of species: tertiary current distribution, Nernst-Planck

- ▲  Tertiary Current Distribution, Nernst-Planck (*tcd*)
  - Species Charges
  - ▶  Electrolyte Liquid (Feed and Permeate)
    - No Flux
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    - Initial Values Membrane
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  - ▲  Ion Exchange Membrane
    -  Equation View
      - Concentration feed
      - Only convection on permeate
      - Periodic Condition
      - Electrolyte Current Feed
      - Electrolyte Current Permeate
      - Electrolyte Potential in one point

## Boundaries selection: only lateral



▼ Equation

Show equation assuming:

Study 1: Flow and Solutes 2D, Stationary ▼

$$-\mathbf{n} \cdot (\mathbf{J}_i + \mathbf{u}c_i) = 0$$

▼ Convection

Include

# Transport of species: tertiary current distribution, Nernst-Planck

## ▲ Tertiary Current Distribution, Nernst-Planck (*tcd*)

Species Charges

▷  Electrolyte Liquid (Feed and Permeate)

No Flux

Insulation

Initial Values Membrane

Initial Values Feed

Initial Values Permeate

▲  Ion Exchange Membrane

$\frac{\partial u}{\partial t} = f$  Equation View

Concentration feed

Only convection on permeate

Periodic Condition

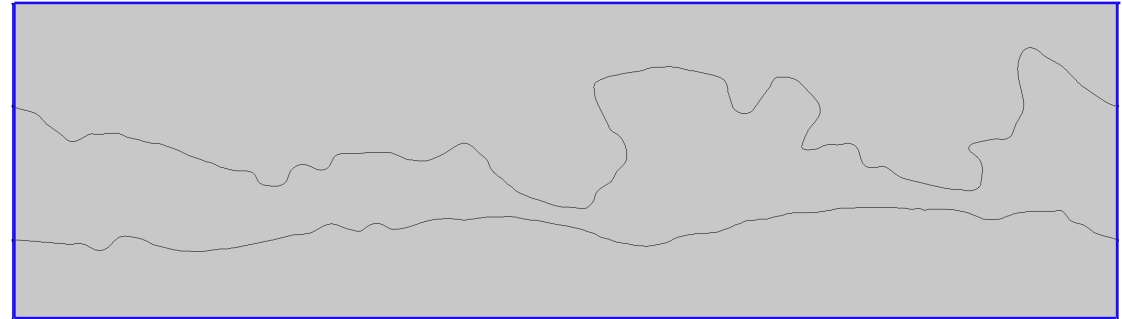
Electrolyte Current Feed

Electrolyte Current Permeate

Electrolyte Potential in one point



Overridden by

**Boundaries selection: external boundaries**

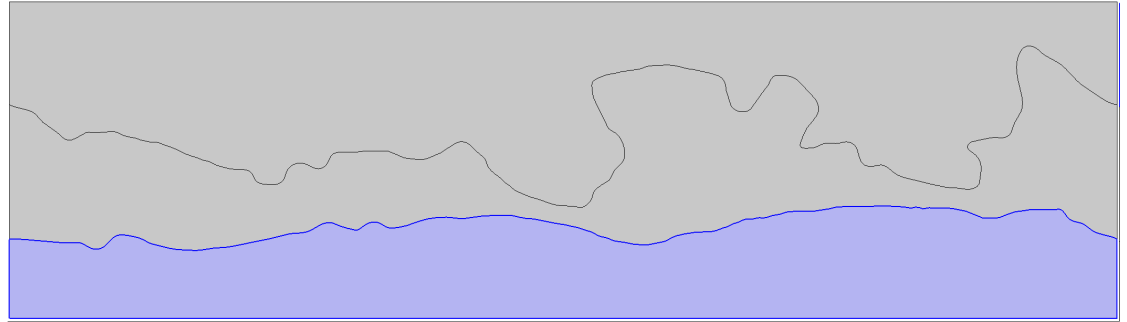




# Transport of species: tertiary current distribution, Nernst-Planck


- ▲  Tertiary Current Distribution, Nernst-Planck (*tcd*)
  - Species Charges
  - ▶  Electrolyte Liquid (Feed and Permeate)
    - No Flux
    - Insulation
    - Initial Values Membrane
    - Initial Values Feed
    - Initial Values Permeate
  - ▲  Ion Exchange Membrane
    -  Equation View
    - Concentration feed
    - Only convection on permeate
    - Periodic Condition
    - Electrolyte Current Feed
    - Electrolyte Current Permeate
    - Electrolyte Potential in one point


**Domain selection:** all domains  
Imposed values on each domain





# Transport of species: tertiary current distribution, Nernst-Planck

## Tertiary Current Distribution, Nernst-Planck (tcd)

 Species Charges


 Electrolyte Liquid (Feed and Permeate)

 No Flux


 Insulation

 Initial Values Membrane

 Initial Values Feed

 Initial Values Permeate


 Ion Exchange Membrane


 Equation View


 Concentration feed

 Only convection on permeate

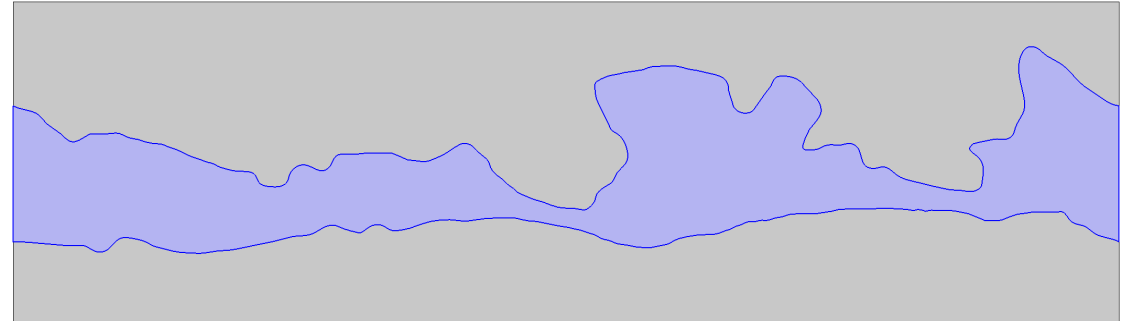
 Periodic Condition

 Electrolyte Current Feed

 Electrolyte Current Permeate

 Electrolyte Potential in one point

## Domain selection: membrane



### Ion Exchange Membrane Properties

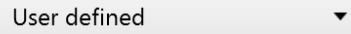
Fixed space charge:

$\rho_{\text{fix}}$   C/m<sup>3</sup>

Apply Donnan boundary conditions

### Convection

Velocity field:

|                             |   |     |
|-----------------------------|---|-----|
| $u \cdot K_f = U \cdot 0.2$ | x | m/s |
| $v \cdot K_f = V \cdot 0.2$ | y |     |

### Diffusion

Material:

Diffusion coefficient:

$D_{c\_cl}$

$D_{cl} \cdot \epsilon_{pse} \cdot K_f = 2.032E-11$  m<sup>2</sup>/s

Diffusion coefficient:

$D_{c\_na}$

$D_{cl} \cdot \epsilon_{pse} \cdot K_f = 1.334E-11$  m<sup>2</sup>/s

# Transport of species: tertiary current distribution, Nernst-Planck

## ▲ 🔗 Tertiary Current Distribution, Nernst-Planck (*tcd*)

D Species Charges

▷ D Electrolyte Liquid (Feed and Permeate)

D No Flux

D Insulation

D Initial Values Membrane

● Initial Values Feed

● Initial Values Permeate

▲ D Ion Exchange Membrane

∂u/∂t = f Equation View

○ Concentration feed

○ Only convection on permeate

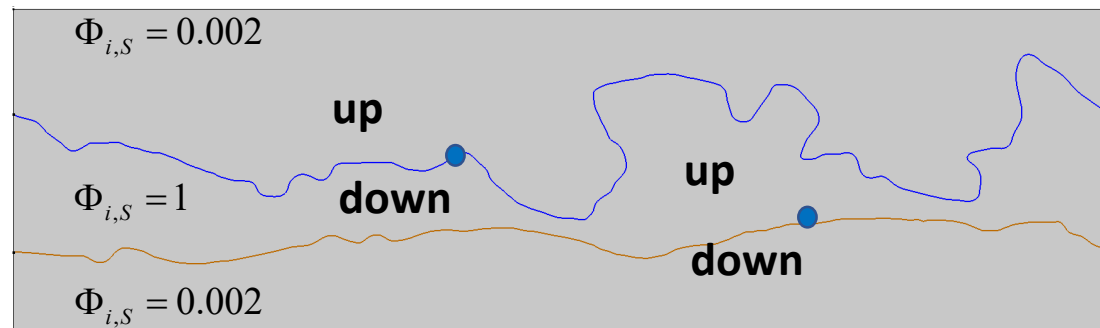
○ Periodic Condition

○ Electrolyte Current Feed

○ Electrolyte Current Permeate

○ Electrolyte Potential in one point

## Constraint on the interface (boundary) feed/active layer



● Concentration on the boundary (feed/active layer)

$$\text{up} \left( \alpha_i \Phi_{i,S} \exp \left( - \frac{z_i F}{RT} (\varphi_M - \varphi_W) \right) \right) \Big|_{\text{up}} - \text{down} \left( c_i \Phi_{i,S} \exp \left( \frac{z_i F}{RT} (\varphi_M - \varphi_W) \right) \right) \Big|_{\text{down}}$$

# Transport of species: tertiary current distribution, Nernst-Planck

## ▲ Tertiary Current Distribution, Nernst-Planck (*tcd*)

Species Charges

▷  Electrolyte Liquid (Feed and Permeate)

No Flux

Insulation

Initial Values Membrane

Initial Values Feed

Initial Values Permeate

▲  Ion Exchange Membrane

Equation View

Concentration feed

Only convection on permeate

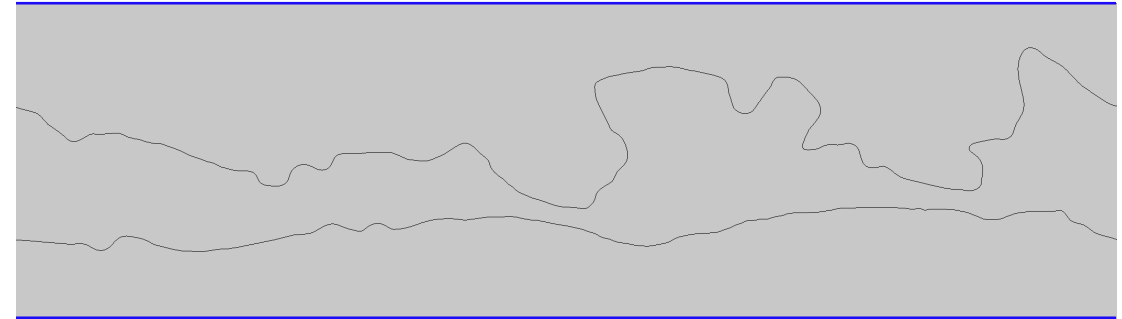
Periodic Condition

Electrolyte Current Feed

Electrolyte Current Permeate

Electrolyte Potential in one point

## Boundary selection: inlet and outlet



Equation

Show equation assuming:

Study 1: Flow and Solutes 2D, Stationary soli ▼

$c_j = c_{0,j}$

Concentration

Species  $c_{cl}$

$c_{0,c_{cl}}$   mol/m<sup>3</sup>

# Transport of species: tertiary current distribution, Nernst-Planck

## ▲ Tertiary Current Distribution, Nernst-Planck (*tcd*)

Species Charges

▷  Electrolyte Liquid (Feed and Permeate)

No Flux

Insulation

Initial Values Membrane

Initial Values Feed

Initial Values Permeate

▲  Ion Exchange Membrane

$\frac{\partial u}{\partial t} = f$  Equation View

Concentration feed

Only convection on permeate

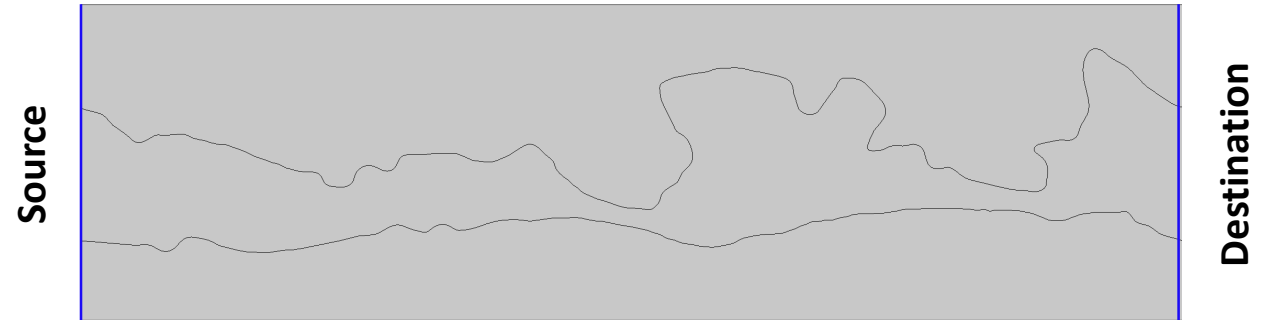
Periodic Condition

Electrolyte Current Feed

Electrolyte Current Permeate

Electrolyte Potential in one point

## Boundary selection: only lateral



### Equation

Show equation assuming:

Study 1: Flow and Solutes 2D, Laminar Flow

$$\phi_{src} = \phi_{dst}$$

$$c_{i,src} = c_{i,dst}$$

$$-\mathbf{n}_{src} \cdot (\mathbf{J}_i + \mathbf{u}c_i)_{src} = \mathbf{n}_{dst} \cdot (\mathbf{J}_i + \mathbf{u}c_i)_{dst}$$

### Periodic Condition



Apply for electrolyte phase

Potential difference:

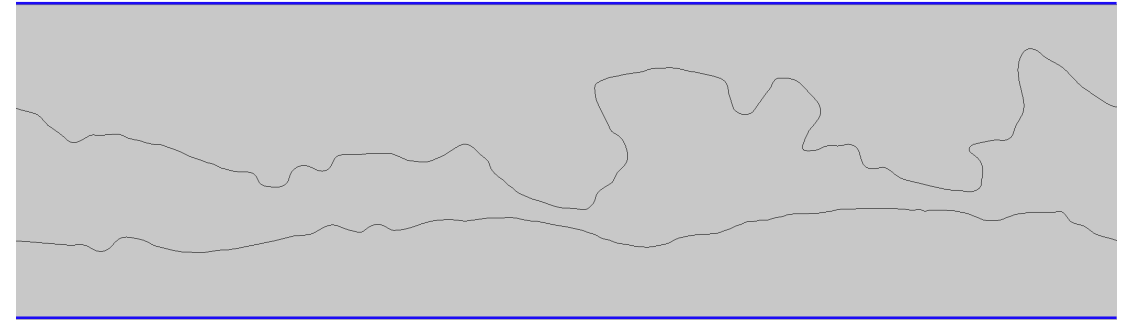
$$\phi_{l,src} - \phi_{l,dst} = 0$$

Apply for electrode phase

# Transport of species: tertiary current distribution, Nernst-Planck

- ▲  Tertiary Current Distribution, Nernst-Planck (*tcd*)
  - Species Charges
  - ▶  Electrolyte Liquid (Feed and Permeate)
    - No Flux
    - Insulation
    - Initial Values Membrane
    - Initial Values Feed
    - Initial Values Permeate
  - ▲  Ion Exchange Membrane
    -  Equation View
      - Concentration feed
      - Only convection on permeate
      - Periodic Condition
      - Electrolyte Current Feed
      - Electrolyte Current Permeate
      - Electrolyte Potential in one point

## Boundary selection: feed and permeate



▼ Equation

Show equation assuming:

Study 1: Flow and Solutes 2D, Laminar Flow ▼

$$-\int_{\partial\Omega} \mathbf{i}_i \cdot \mathbf{n} dl = i_{i,average} \int_{\partial\Omega} dl$$

▼ Electrolyte Current

Average current density ▼

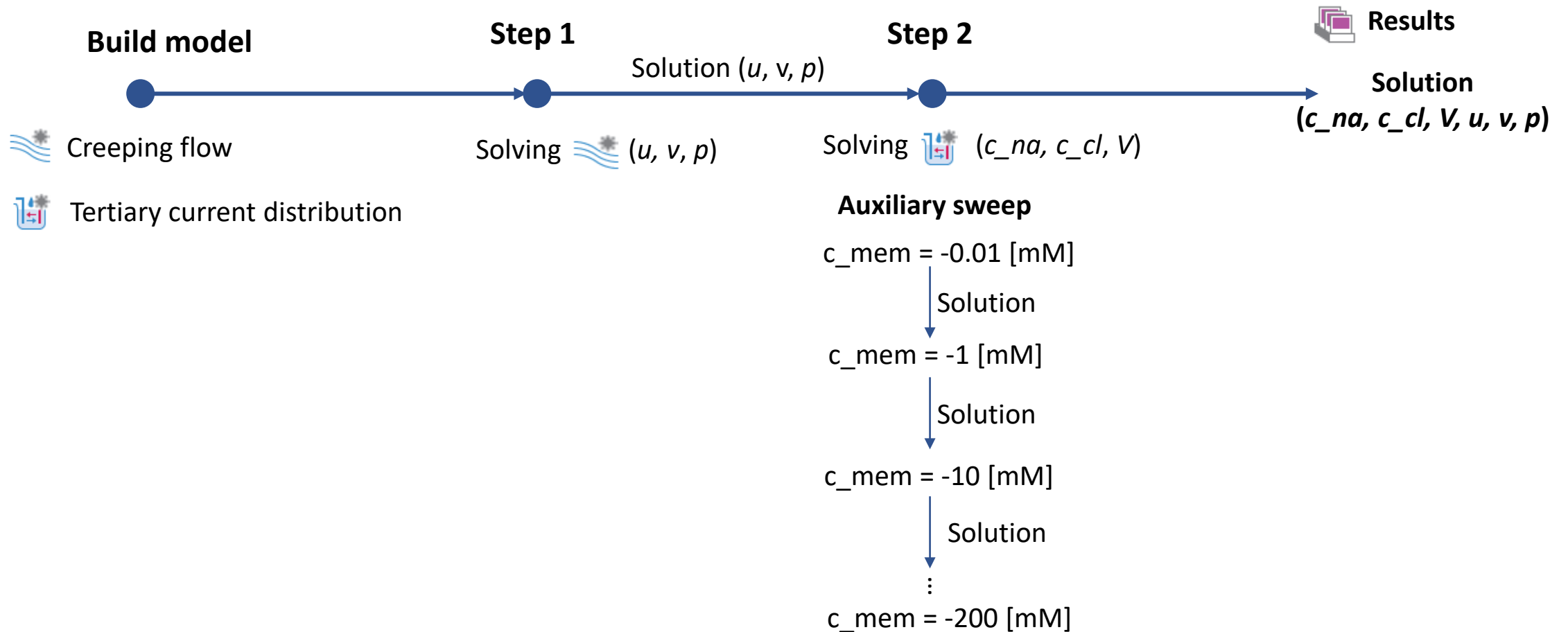
Inward electrolyte current density:

$i_{i,average}$   A/m<sup>2</sup>

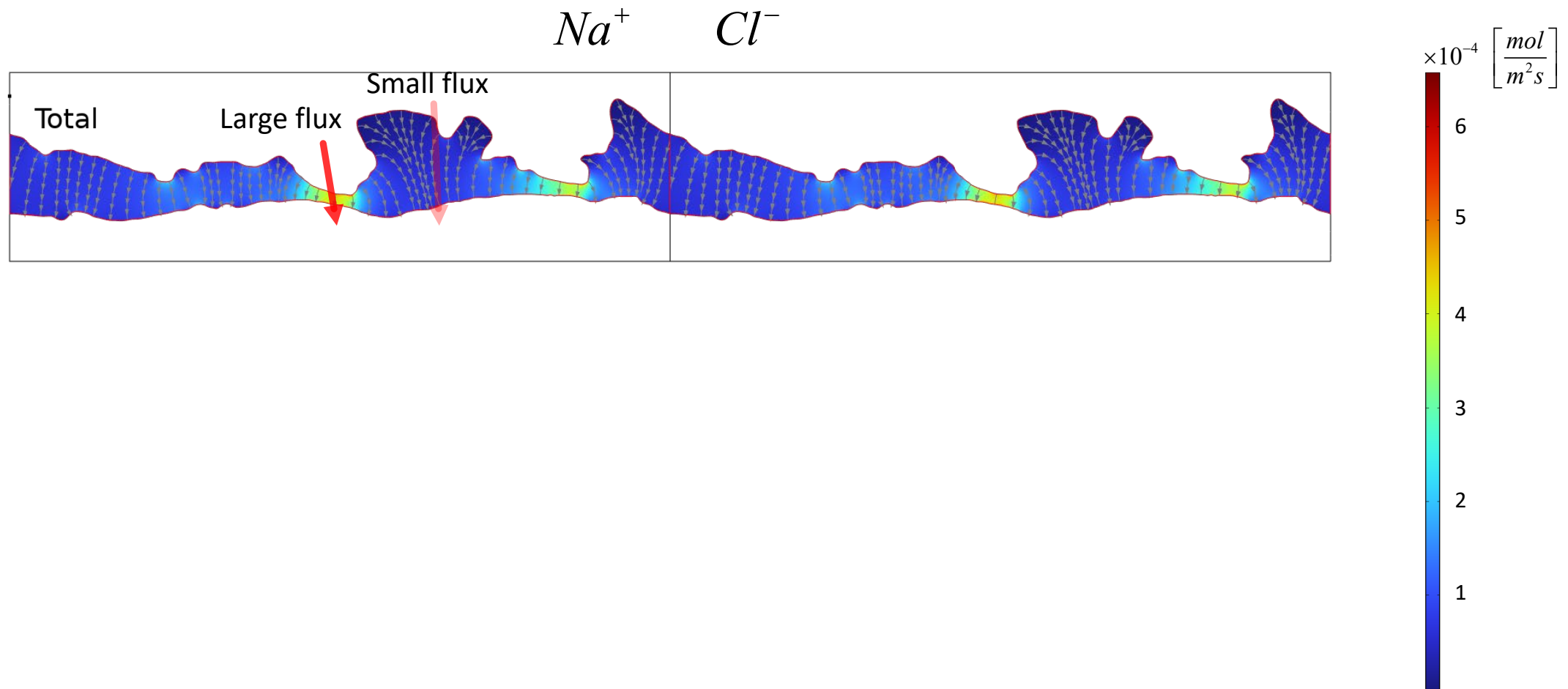
Boundary electrolyte potential initial value:

$\phi_{l,bnd,init}$   V

# Solution strategy

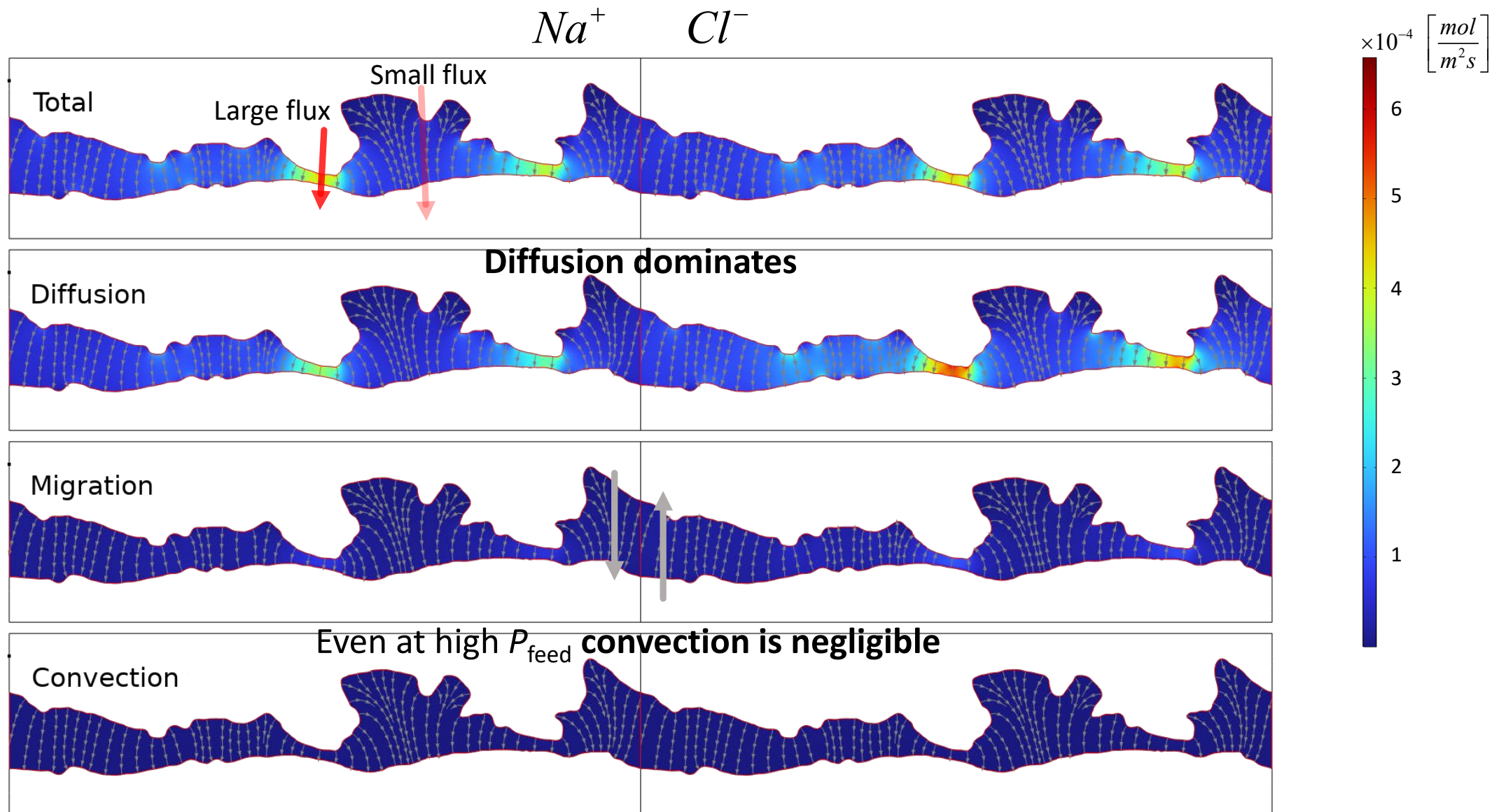


# 2-D Model. Results - Fluxes in uncharged membrane (-0.01 mM)



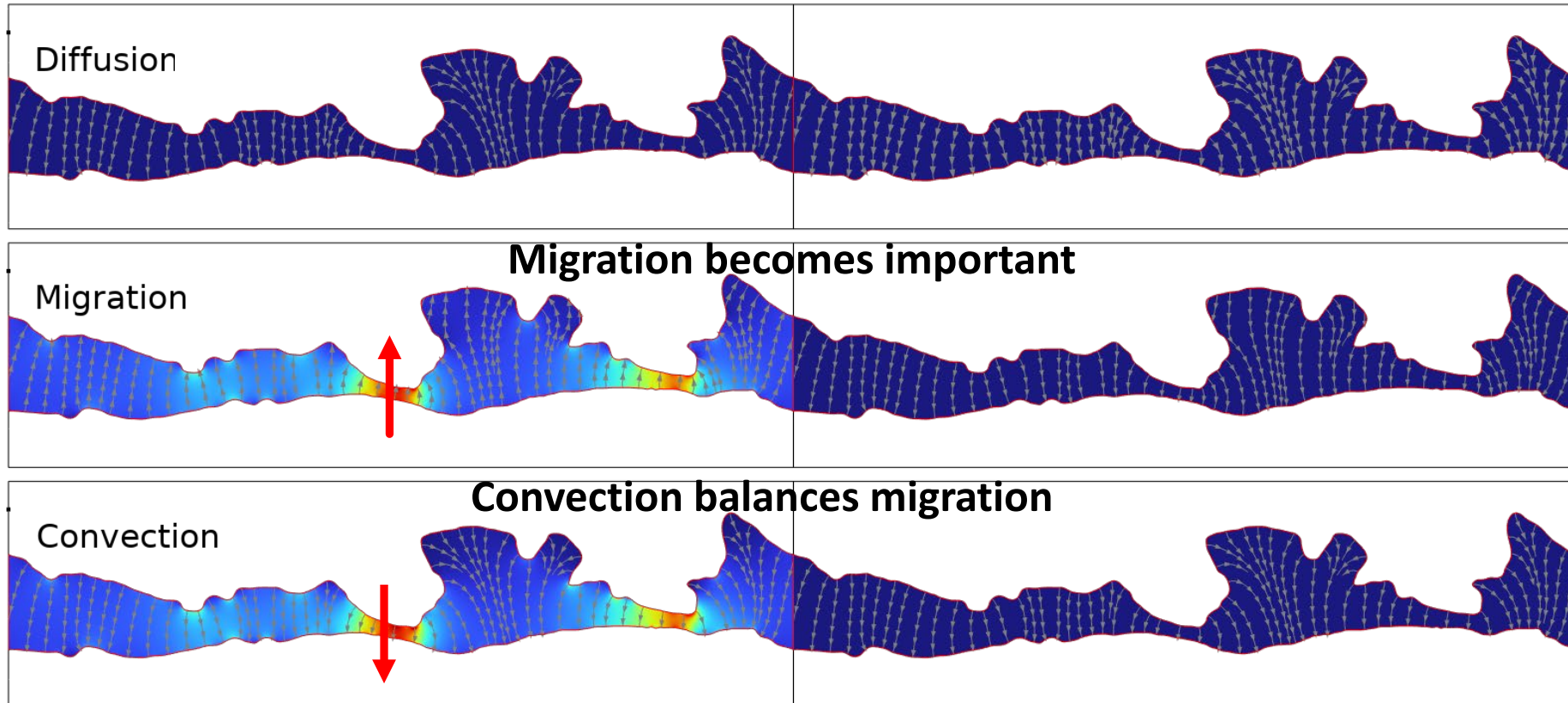
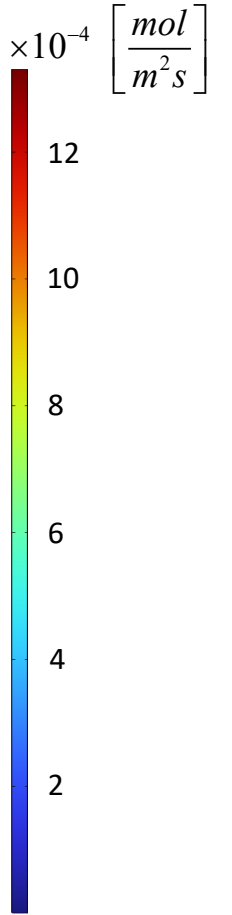


# 2-D Model. Results - Fluxes in uncharged membrane (-0.01 mM)

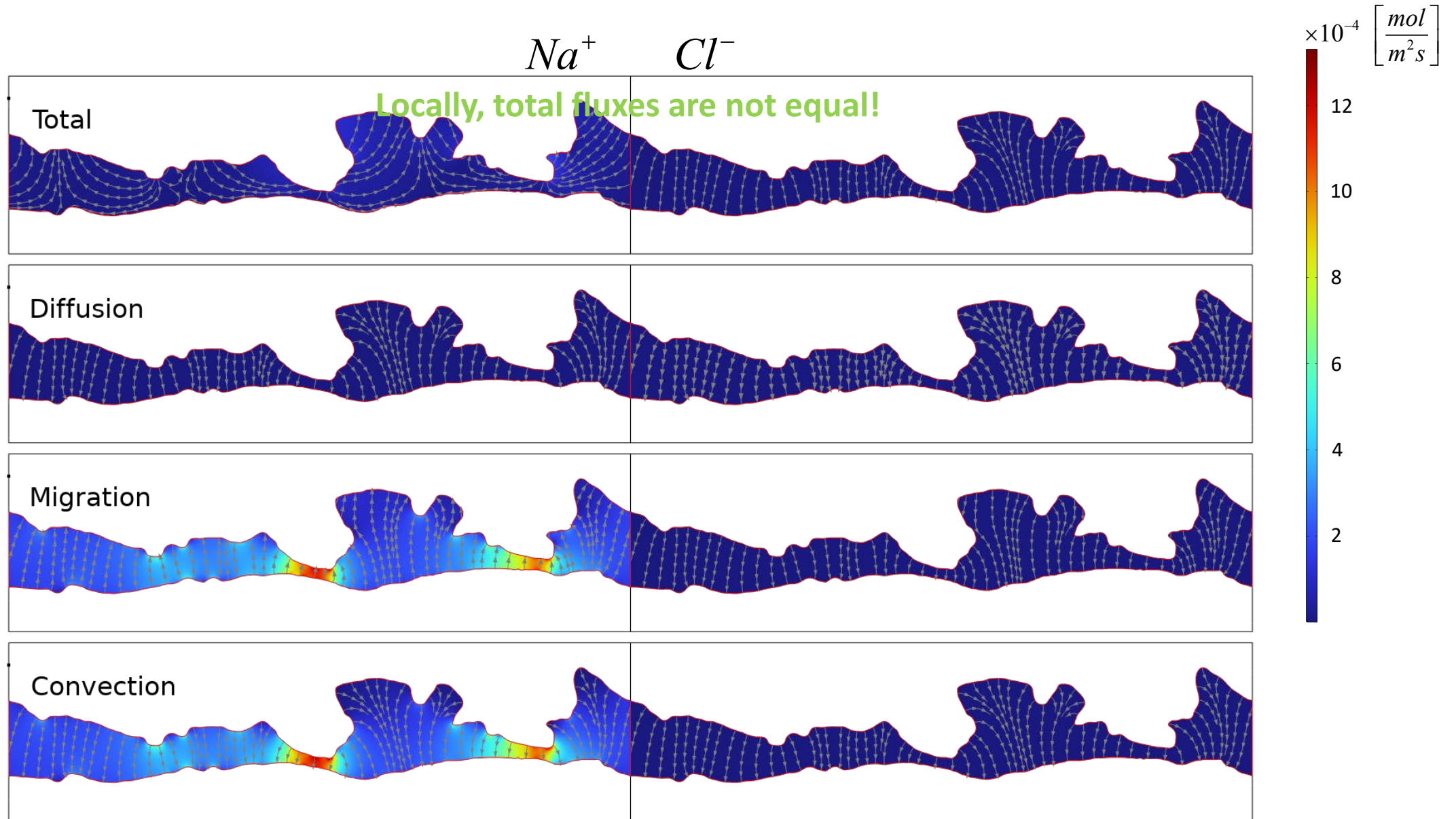


# 2-D Model. Results - Fluxes in charged membrane (-200 mM)

$Na^+$        $Cl^-$

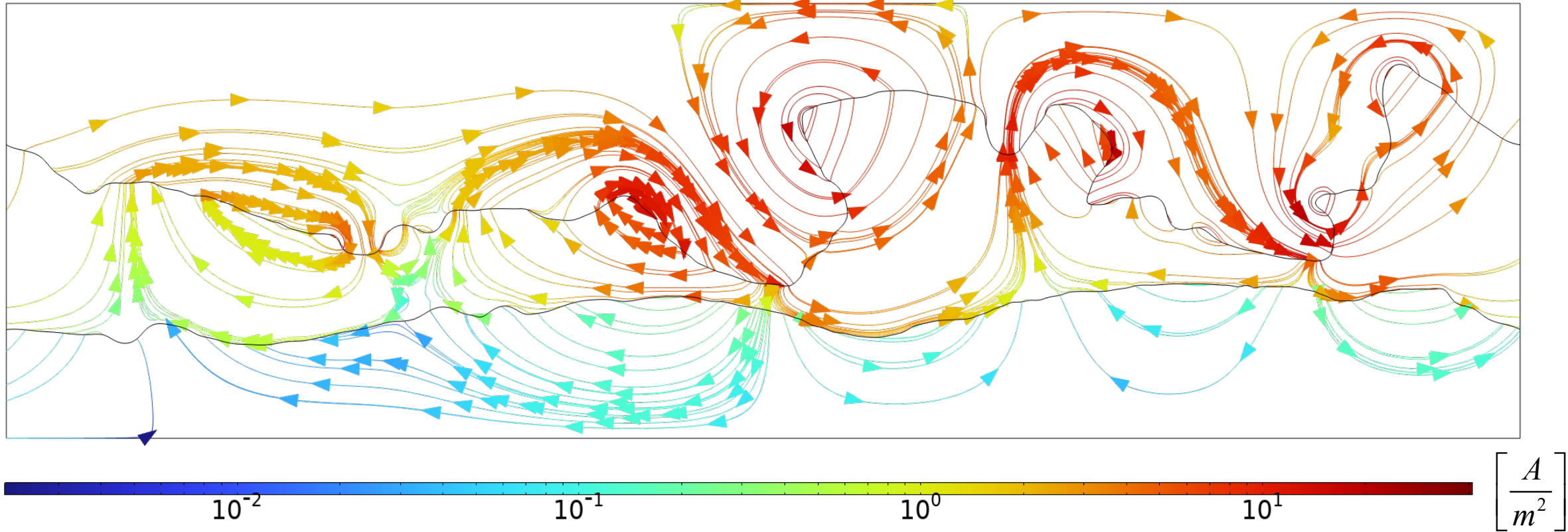


# 2-D Model. Results - Fluxes in charged membrane (-200 mM)



# 2-D Model. Ionic current density

There are ionic currents in the 2-D membrane



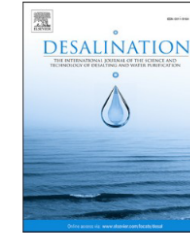
Closed current circuits (insulation)



Contents lists available at ScienceDirect

# Desalination

journal homepage: [www.elsevier.com/locate/desal](http://www.elsevier.com/locate/desal)



## Two-dimensional model of ion transport in composite membranes active layers with TEM-scanned morphology

Fernan David Martinez-Jimenez<sup>a,\*</sup>, Valentina-Elena Musteata<sup>b</sup>, Santiago Cespedes-Zuluaga<sup>a</sup>, Bastiaan Blankert<sup>a</sup>, Cristian Picioreanu<sup>a</sup>

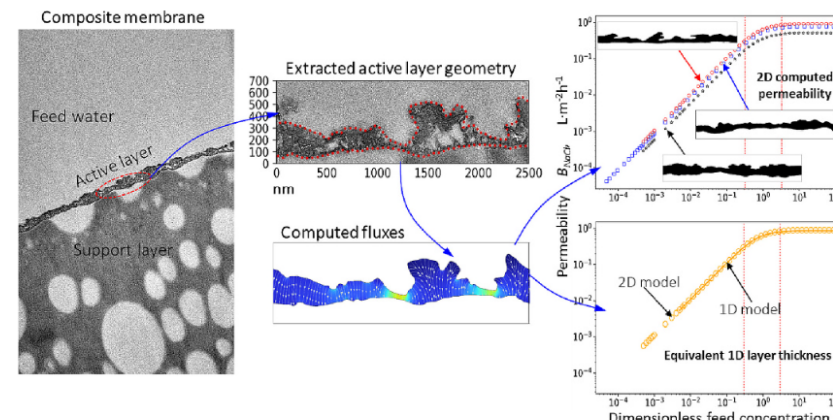
<sup>a</sup> Environmental Science & Engineering Program (EnSE), Biological and Environmental Science and Engineering Division (BESE) and Water Desalination and Reuse Center (WDRC), King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia

<sup>b</sup> KAUST Core Labs, King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia

### HIGHLIGHTS

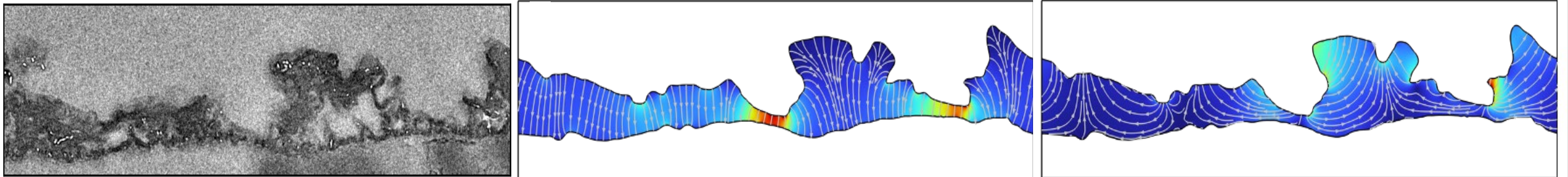
- A 2D solution-friction model reveals new effects of active layer on ion permeability.
- Ionic current loops may develop inside and around the active layer.
- Different transport mechanisms are dominant in different conditions.
- An equivalent 1D active layer thickness leads to the same permeability as in 2D.
- The equivalent thickness can be computed from images of the active layer.

### GRAPHICAL ABSTRACT



# Key messages

1. The response of the 2-D model to variations in flux and salinity can be represented by a 1-D model using an appropriate *equivalent membrane thickness*.
2. We provided a method to compute the *equivalent membrane thickness* from images of membrane active layer.
3. The 2-D model revealed *the possibility of circular ionic currents*.



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# 6<sup>TH</sup> PHYSICS OF MEMBRANE PROCESSES WORKSHOP

November 13 - 16 , 2023

King Abdullah University of Science and Technology  
(KAUST), Saudi Arabia



جامعة الملك عبد الله  
للعلوم والتقنية  
King Abdullah University of  
Science and Technology



Scan the code to learn more about the workshop

[wdrconferences@kaust.edu.sa](mailto:wdrconferences@kaust.edu.sa)

[fernandmartinez@kaust.edu.sa](mailto:fernandmartinez@kaust.edu.sa)

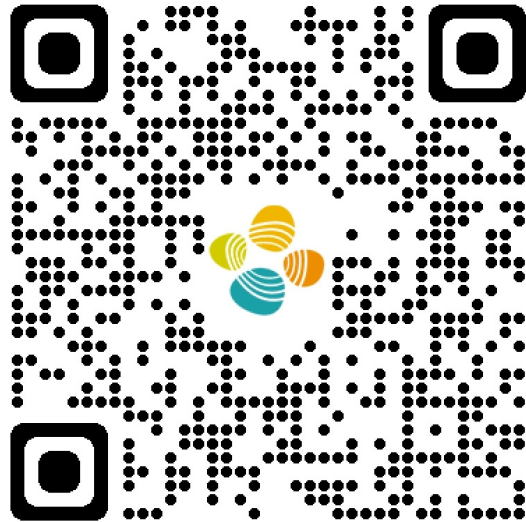


- Fundamentals of membrane transport processes
- Theory and computer applications in **COMSOL**
- Experimental aspects of transport phenomena in membranes, from small-scale to system-level

# Thank you

fernan.martinez@kaust.edu.sa

## Article

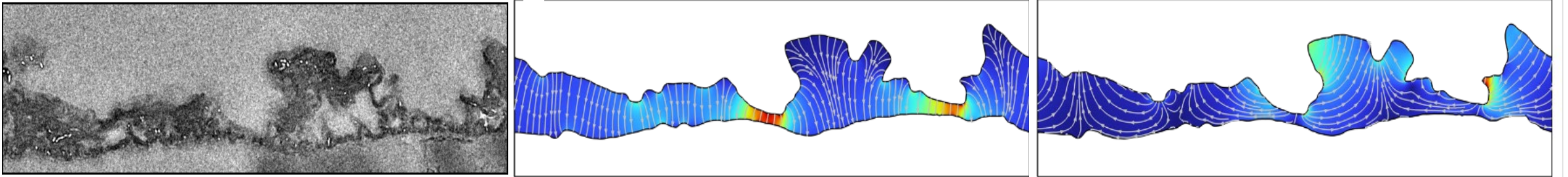


## PMP 6<sup>th</sup> conference





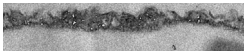

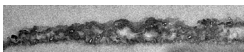

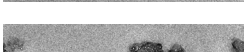




# SUPPLEMENTARY INFORMATION



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# Ideal 1D and equivalent active layer thickness $L_3$ , for all active layer geometries

| Active layer  | Geometry | Average<br>2D<br>thickness<br>$L_1$ , nm | Relative<br>standard<br>deviation<br>of $L_1$ , $\sigma/L_1$ | Reference<br>1D<br>thickness<br>$L_B$ , nm | Reference<br>1D<br>thickness<br>$L_A$ , nm | Equivalent<br>thickness<br>$L_3$ , nm |
|---|----------|--|--|--|--|---------------------------------------|
|    | <b>a</b> | 181                                      | 0.29   | 149  | 154  | 149                                   |
|    | <b>b</b> | 175                                      | 0.35   | 125  | 130  | 131                                   |
|    | <b>c</b> | 160                                      | 0.27   | 128  | 132  | 128                                   |
|    | <b>d</b> | 270                                      | 0.27   | 224  | 231  | 220                                   |
|    | <b>e</b> | 198                                      | 0.23   | 165  | 170  | 165                                   |
|    | <b>f</b> | 250                                      | 0.38   | 177  | 183  | 178                                   |
|   | <b>g</b> | 221                                      | 0.51   | 129  | 133  | 129                                   |
|  | <b>h</b> | 324                                      | 0.40   | 239  | 247  | 239                                   |
|  | <b>i</b> | 214                                      | 0.34   | 179  | 185  | 180                                   |

# Feed and permeate:

$$\nabla \cdot \left( -D_i \nabla c_i - \frac{z_i D_i F}{RT} c_i \nabla \varphi + \mathbf{u} c_i \right) = 0$$

$$\mathbf{J}_i = -D_i \nabla c_i - \frac{z_i D_i F}{RT} c_i \nabla \varphi + \mathbf{u} c_i$$

$$\mathbf{i} = F \sum_i z_i \mathbf{J}_i$$

$$\sum_i z_i c_i = 0$$

# Membrane:

$$\nabla \cdot \left( -D_{i,eff} \nabla c_i - \frac{z_i D_{i,eff} F}{RT} c_i \nabla \varphi + K_f \mathbf{u} c_i \right) = 0$$

$$\mathbf{J}_i = -D_{i,eff} \nabla c_i - \frac{z_i D_{i,eff} F}{RT} c_i \nabla \varphi + K_f \mathbf{u} c_i$$

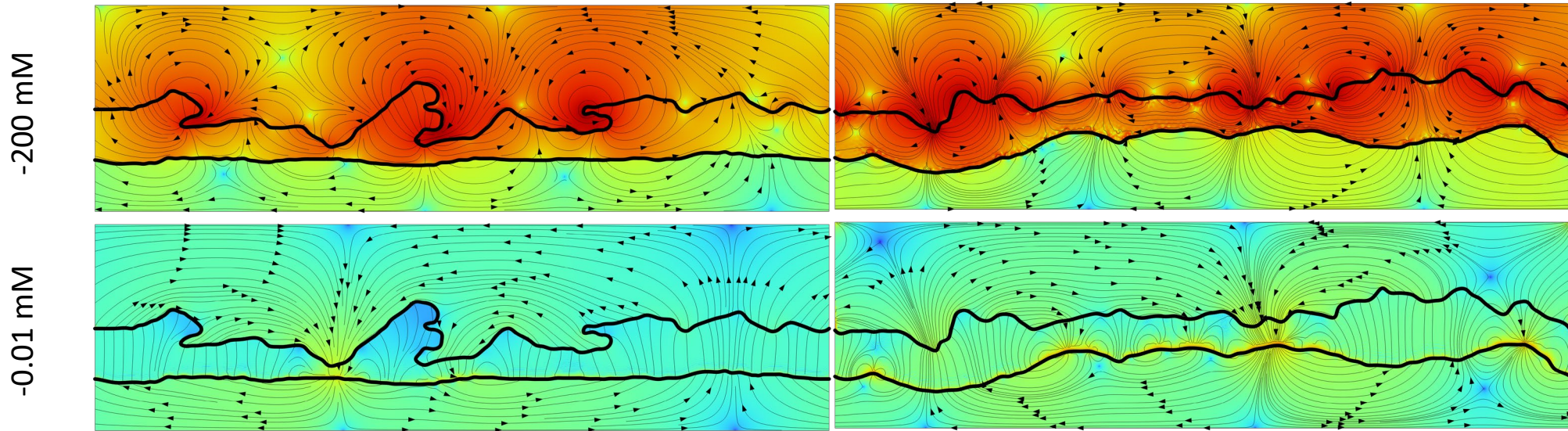
$$\mathbf{i} = F \sum_i z_i \mathbf{J}_i$$

$$z_M c_M + \sum_i z_i c_i = 0$$

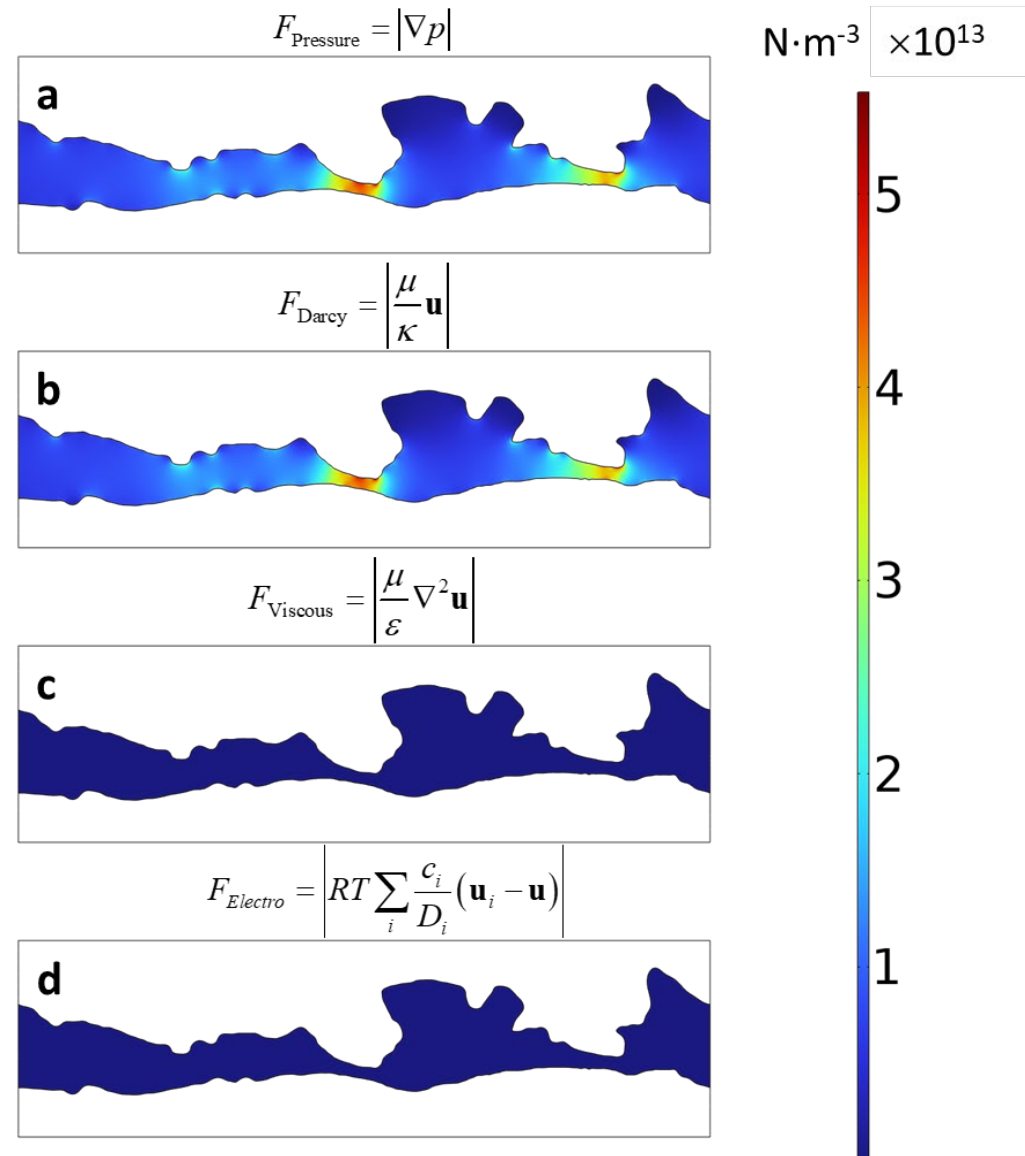
$$\underbrace{\frac{1}{\varepsilon^2} \rho (\mathbf{u} \cdot \nabla) \mathbf{u}}_{\text{Inertial force}} = - \underbrace{\nabla p}_{\text{Pressure force}} + \underbrace{\frac{\mu}{\varepsilon} \nabla^2 \mathbf{u}}_{\text{Viscous force}} - \underbrace{\frac{\mu}{\kappa} \mathbf{u}}_{\text{Darcy force}} + \underbrace{RT \sum_i \frac{c_i}{D_i} (\mathbf{u}_i - \mathbf{u})}_{\text{Electroosmotic force}}$$

$$\frac{1}{\varepsilon^2} \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{\mu}{\varepsilon} \nabla^2 \mathbf{u} - \frac{\mu}{\kappa} \mathbf{u} + RT \sum_i \frac{c_i}{D_i} (\mathbf{u}_i - \mathbf{u})$$

# 2-D Model. Ionic current density



# Comparison between the magnitudes of forces affecting water permeation through the active layer:



## 2-D Model. Water transports in the membrane. SD or SF?

SD model is equivalent to the Darcy equation when the osmotic pressure gradient is negligible

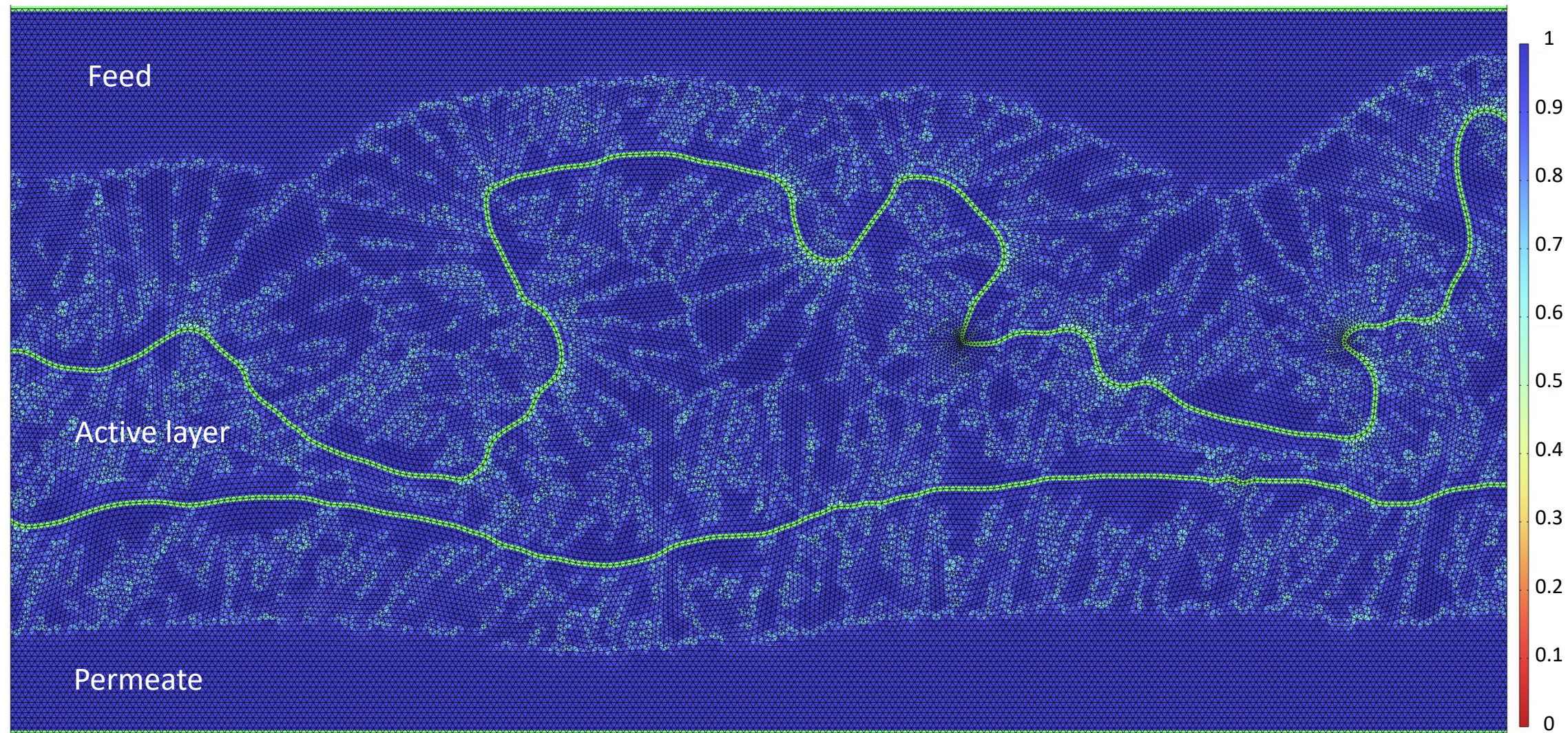
$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p \quad \Rightarrow \quad J_W \propto \frac{dp}{dx}$$

$$J_W = A(\Delta p - \sigma \Pi) \quad \Rightarrow \quad J_W \propto \frac{dp}{dx}$$


strong water/membrane partitioning



Figure SI 4



# Model implementation. Transport of water: creeping flow

- ▲  Creeping Flow (*spf*)
  - Feed and permeate properties
  - Initial Values 1
  - Wall 1
- ▲  Porous Medium
  - Fluid
  - Porous Membrane
  - Inlet pressure feed
  - Outlet pressure permeate
  - Periodic Flow Condition

**Domain selection:** all domains  
Imposed values on each domain

