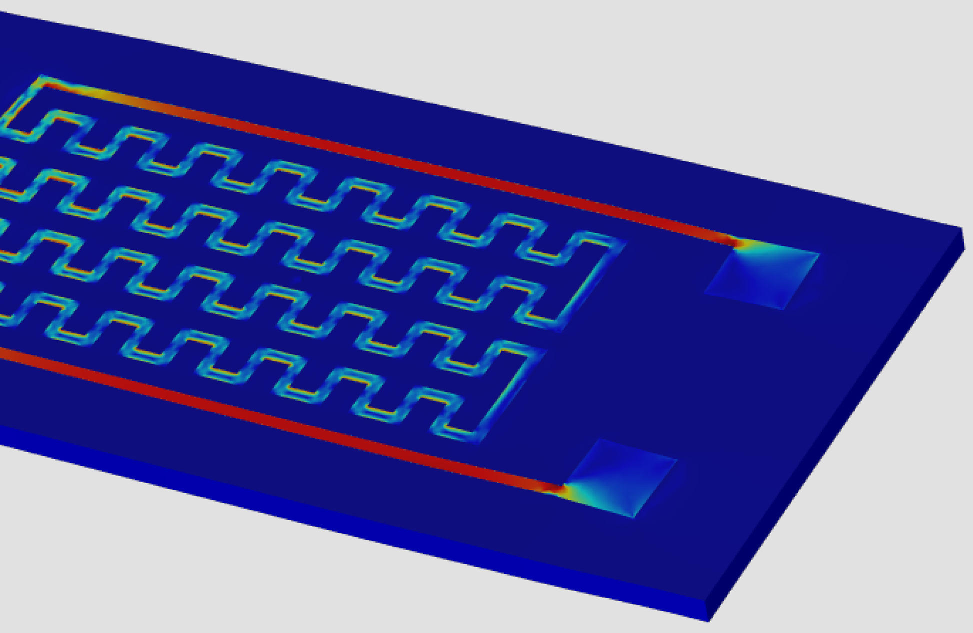

# Numerical Analysis of Hyperelastic Material Behavior Towards Shape-Adjustable Sensor Devices


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## Abstract


In this work, the numerical analysis of stretchable thermoplastic polyurethane (TPU) substrates with integrated copper traces serving as resistive or rather capacitive sensor elements is presented. Fundamentals of phenomenological hyperelastic material models with emphasis on the 5 parameter Mooney-Rivlin approach to describe the material behavior of TPU best possible are discussed in a first instance. Capabilities for shape optimization of embedded copper traces are additionally studied by means of COMSOL Multiphysics® in consideration of TPU strain rates up to 60%. Based on these findings, first designs for shape-adjustable resistive and capacitive sensor devices are successfully realized and modeled by FEM simulations.


## Fundamentals of Hyper-Elastic Material Models

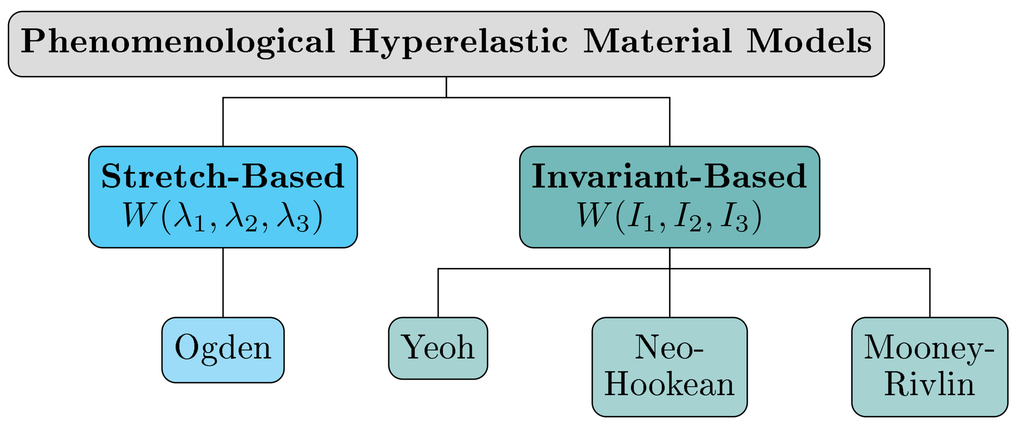


**Fig. 1:** Classification of phenomenological hyperelastic material models. Adapted from [1,2].

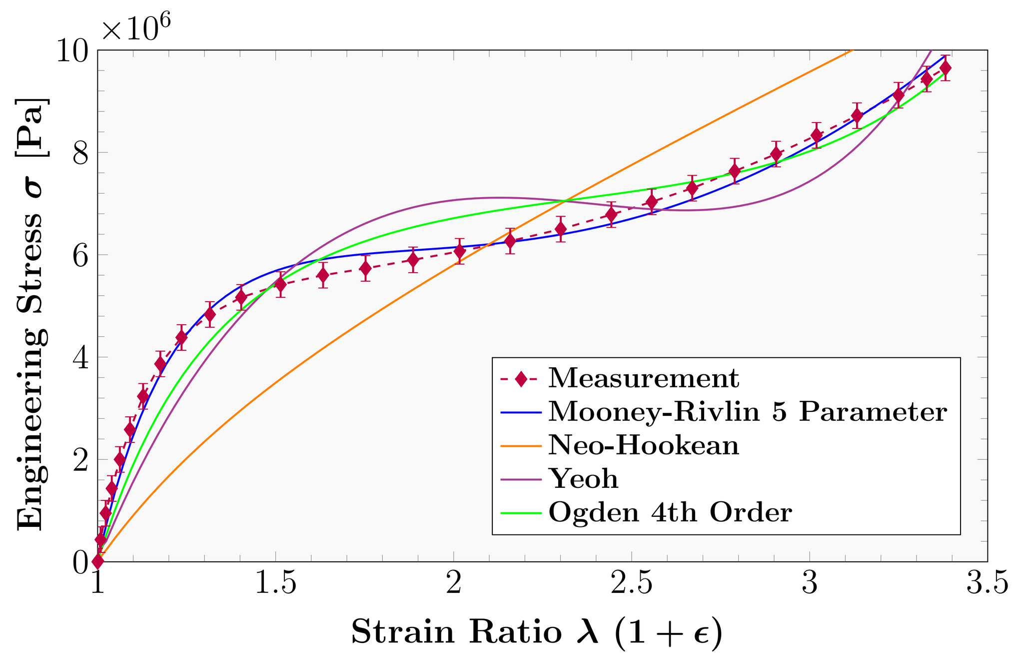


**Fig. 2:** Comparison of hyperelastic material models for description of TPU.

## Governing Equations of the 5 P Mooney-Rivlin Model

**Strain energy density function $W_{MR}$:**

$$\begin{aligned} W_{MR} = & \; C_{10}(I_1-3)+C_{01}(I_2-3) \\ & +C_{20}(I_1-3)^2+C_{02}(I_2-3)^2 \\ & +C_{11}(I_1-3)(I_2-3) \\ & +\frac{\kappa_{MR}}{2}(J_{el}-1)^2 \end{aligned}$$

**Expression for the relating invariants $I$:**

$$I_1=\left(\lambda^2+\frac{2}{\lambda}\right) \quad and \quad I_2=\left(2\lambda+\frac{1}{\lambda^2}\right)$$

**Derived stress-strain relation $\sigma_{MR}$:**

$$\begin{aligned} \sigma_{MR} = 2\left(1-\lambda^{-3}\right) \Big( & \lambda C_{10}+2C_{20}\lambda(I_1-3) \\ & +C_{11}\lambda(I_2-3)+C_{01}+2C_{02}(I_2-3) \\ & +C_{11}(I_1-3)\Big) \end{aligned}$$

## Shape Optimization of Functional Sensor Elements

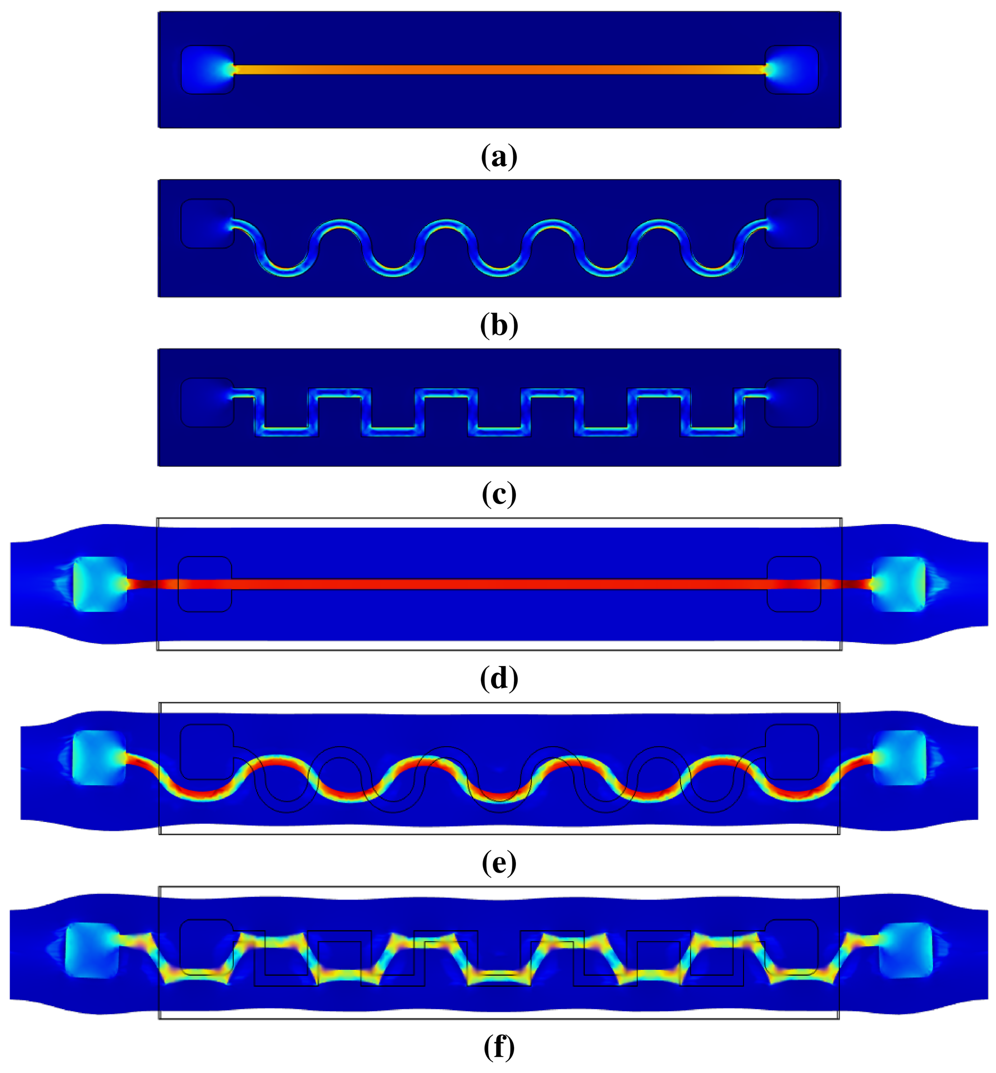


**Fig. 3:** Shape optimization of TPU with conductive copper traces: (a) Straight copper path, (b) meanders of 180° (U-shaped) and (c) rectangular traces of metallic structures in their initial state, (d)-(f) copper trace geometries at 40% strain.

## Electrical Characterization of Sensor Elements

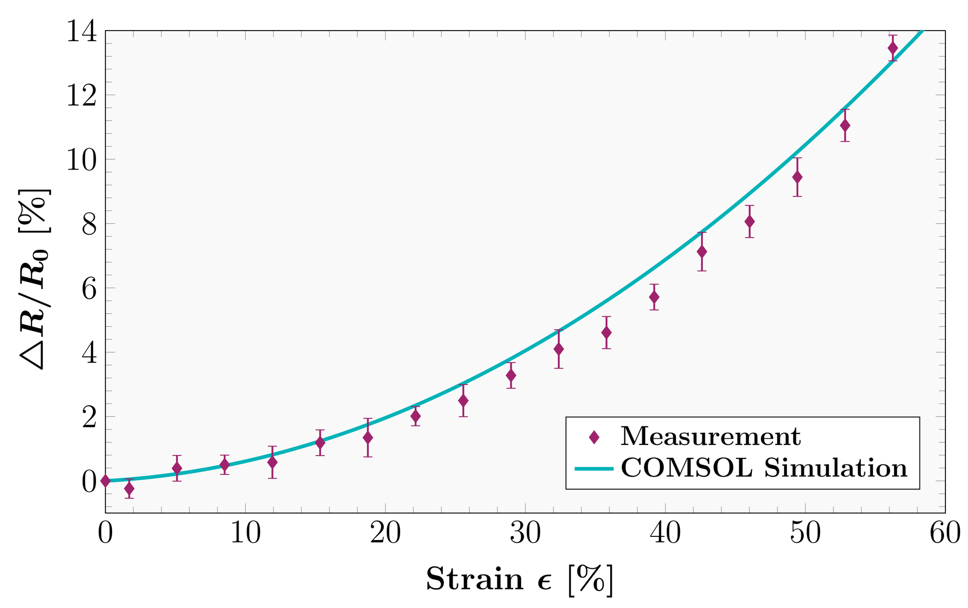


**Fig. 4:** Relative change in resistance $\triangle R/R_0$ of the strain gauge: Comparison of simulation and measurement results for strain rates up to 60%.

## Initial Resistances & Stresses of the Copper Geometries

| Geometry | | $R_{0,norm.}$ | $\sigma_M$ |
|---|---|---|---|
| Straight | | 100% | 100% |
| Meander 180° | | 157% | 90% |
| Rectangular | | 291% | 26% |

## FEM Modeling of Sensors

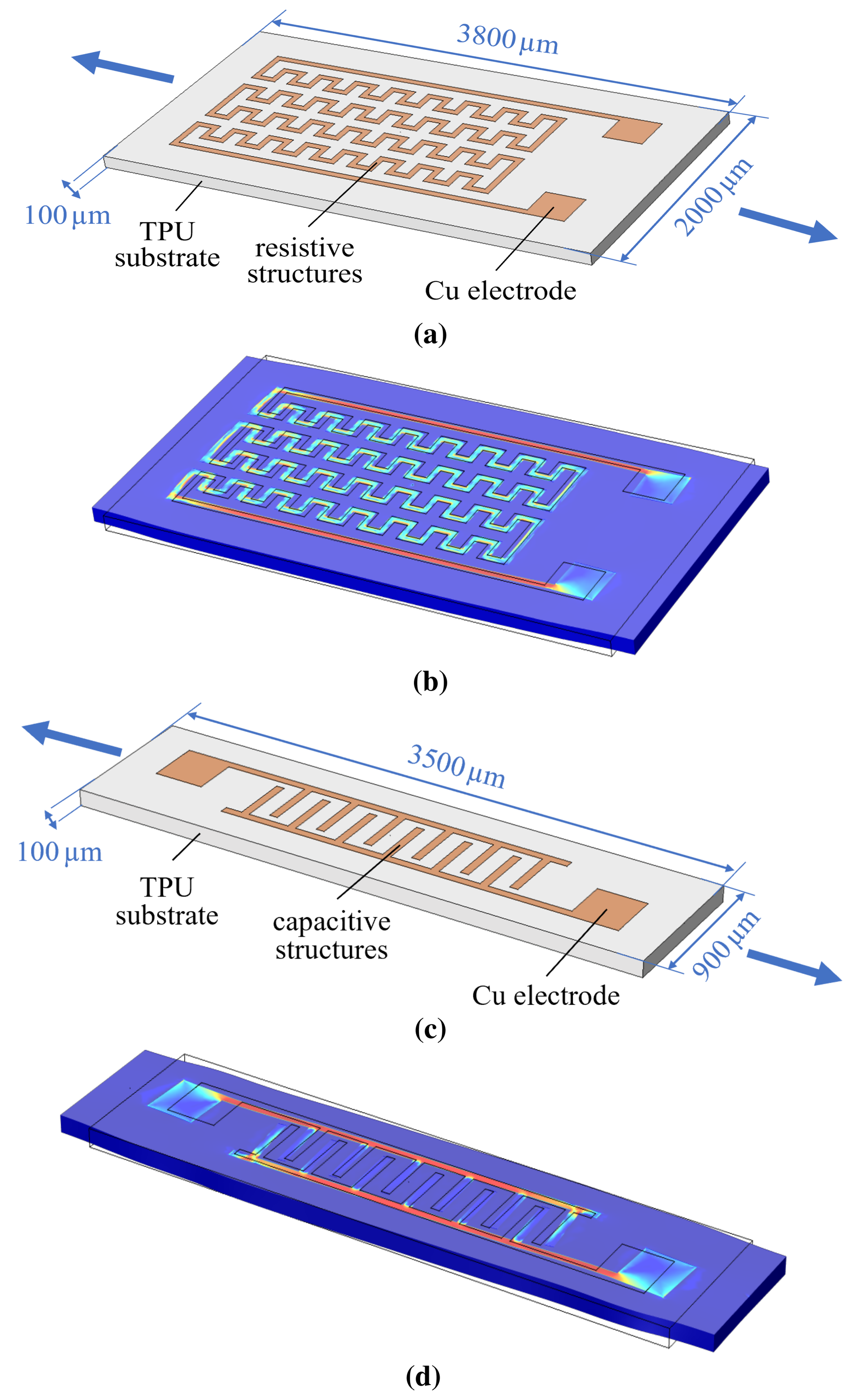


**Fig. 5:** Shape-adjustable sensors: (a) Resistive sensor with 100 µm thick TPU, (b) relating FEM model at 15% strain, (c) capacitive sensor design and (d) capacitive sensor at 15% strain.

## Conclusion & Outlook

FEM models of resistive and capacitive sensors based on TPU sheets with functional copper elements are introduced successfully aiming the development of novel sensors for human health monitoring. Different invariant- and stretch-based material models are discussed initially. Moreover, options for shape optimization towards 180° meanders and rectangular copper traces of the devices have been highlighted. In conclusion, the simulations have proven an uniaxial deformation of both sensor versions up to 35% without loss of functionality.

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Excerpt from the Proceedings of the COMSOL Conference 2024 Florence