Nelder Mead simplex algorithm to optimize geometry for maximum objective function value

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Abstract
The design of electromagnetic systems requires a procedure to maximize certain parameters within given constraints. Optimization routines that maximize the function value under these constraints are essential. This design and optimization process can be carried out in a COMSOL optimization study using the Nelder-Mead (NM) simplex algorithm. In this paper, the COMSOL NM simplex optimization routine is used to maximize the coupling coefficient ‘k’ between two inductively coupled coils that constitute wireless charging of electric vehicles. The geometric dimensions that affect ‘k’ are employed as control variables. A frequency domain problem is formulated to design concentric symmetrical circular inductors to achieve the maximum value of ‘k’. The COMSOL NM simplex algorithm is found to return a solution where the geometry parameters yield the highest value of ‘k’ and mutual inductance. This, in turn, results in a higher level of power transferred to the load. The developed design procedure can be used in Inductive Power Transfer (IPT) systems maximizing magnetic coupling while keeping system dimensions within limits. Additionally, this procedure aids in making more informed decisions about a system’s parameters, which would otherwise be difficult and time-consuming.

Keywords: Electromagnetic system design, geometry optimization, maximized objective function, coupling coefficient, mutual inductance, inductive power transfer.

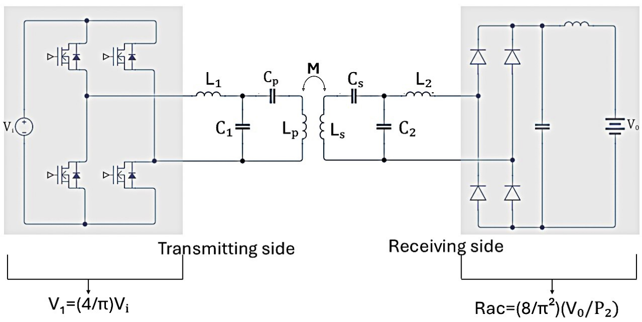
I. Introduction
Optimization techniques and methods are crucial for enhancing problem analysis, refining system design, and boosting overall performance. These techniques are employed across various fields of engineering to improve design procedures. One such example is electromagnetics, where these methods are used to achieve the best design relative to limitations and constraints. Without optimization methods, electromagnetic design relies upon parameter evaluation at each combination of variables. As the number of variables increases, the procedure becomes complicated and time-consuming. This increases the chances of inaccuracy and compounds the amount of required computing power.

Several optimization techniques and algorithms have been developed to address different engineering design problems. For example, gradient-based optimization routines can be used to solve problems where the objective can be defined by a well-established mathematical function/relation between variables and has a well-defined derivative. These routines follow the path of a reduced objective function derivative and can find the solution where the function is optimum. Gradient-based optimization routines mostly find the minimum value of a function by looking for the solution where the higher-order derivative of a function is zero or nearly zero. However, the same procedure is used to find the maximum value of a function by finding the minimum value of the negative objective function. Nevertheless, gradient-based techniques are unreliable when the maximum value of the function is required, and the function derivative cannot be well defined.

Since many electromagnetic systems require the maximum value of certain parameter(s) which do not have a well-defined derivative. Gradient-free heuristic optimization approaches are used to solve such engineering design problems as they are derivative-free. The Nelder Mead (NM) simplex is one such optimization technique [1] and achieves improved objective function value by iteratively replacing the worst point in the control variable space. This is suitable for problems where the objective function is non-smooth. One typical example of such an objective function is when the control variables define geometry dimensions in the electromagnetic system. Unlike other gradient-free optimization routines [2], NM simplex is easy and requires less time computationally.

COMSOL Multiphysics is a powerful and versatile software package that offers a wide range of strengths for Finite Element Analysis (FEA). FEA is a numerical technique that provides an approximate solution to the partial differential equations that are hard to solve analytically. FEA can provide accurate predictions of the behaviour of physical electromagnetic systems. It also helps in optimizing system design and performance. In this paper, COMSOL is used to design inductively coupled coils for the wireless charging of electric vehicles. Optimization is used in the study step. An optimal design is obtained that can achieve maximum coupling ‘k’ between the coils and lead to a higher level of power transfer.

Inductive power transfer (IPT) technology has emerged as a promising solution for wireless charging of EVs [3]. An equivalent circuit diagram of the transmitting and receiving sides of IPT, using DLCC compensation topology [4], is shown in Figure 1(a). An analysis is performed on the first harmonic component of inverter output. The rectifier and load are replaced with equivalent resistance (making a unity power factor receiver). This reduces the circuit to that shown in Figure 1(b).



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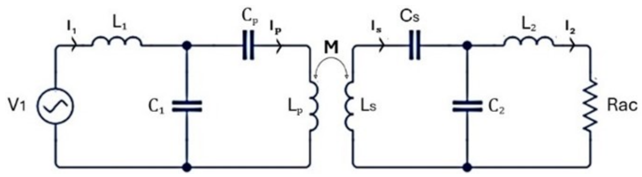


(b)

Figure 1: (a) IPT system with DLCC compensation (b) Equivalent circuit

Power transferred to load 'R_{ac}', in Figure 1, is given by:

$$P = \frac{8V_oV_iM}{\pi^2\omega L_1L_2} \tag{1}$$

V_i is an input voltage, V_o is a output voltage, L_1 is a transmitting IPT coil, L_2 is a receiving IPT coil, and M is a mutual inductance between coils L_1 and L_2. Eq. (1) shows a higher value of mutual inductance 'M' is required to obtain higher power at load. 'M' is directly proportional to the coupling coefficient 'k'. This creates a need to design inductors, L_p and L_s, that can achieve higher 'k'. To maximize 'k' between L_p and L_s coils, the objective function is defined as:

$$k = \frac{\text{Ls coil open-circuit voltage}}{j\omega|\text{Lp coil current}|(\text{Lp coil self-inductance})} \tag{2}$$

In this paper L_p and L_s are designed in an AC-DC module using parameter optimization in the study steps. While 'magnetic field' physics provides a solution for magnetic vector potential, the Nelder Mead (NM) simplex algorithm is used for optimization purposes.

In terms of paper structure, section II explains the NM simplex optimization routine and algorithm flow from the perspective of objective function maximization. Section III details the IPT problem formulation and simulation through COMSOL. Section IV discusses the strengths of NM simplex and its application in electromagnetic system design, while section V provides conclusions.

II. Nelder Mead Simplex Optimization Routine

At the core of NM is a simplex method. Simplex is a standard technique for solving an optimization problem with an objective function and several constraints expressed as inequalities. The inequalities define a polygonal region, and a solution is typically at one of the vertices. Simplex systematically tests vertices as possible solutions. For the maximization of an objective function, simplex provides a systematic search until the feasible solution is identified (one where the objective function is at its maximum). NM performs a simplex routine in which vertices are developed by a reflection, an expansion, a contraction, or a shrink (which rarely occurs in practice) with respect to the best vertex. This is explained in detail next.

Algorithm for function maximization

In NM iterations the worst vertex (y_n) is replaced by a point in the line that connects y_n and y_c,

$$y = y_c + \delta(y_c - y_n)$$

$$y_c = \sum_{i=0}^{n-1} \frac{y_i}{n}$$

(δ is a real number and y_c is the centroid of the best n vertices). The value of δ indicates the type of iteration. When $\delta = 1$, $\delta = 2$, $\delta = \frac{1}{2}$ and $\delta = -\frac{1}{2}$, there is a genuine reflection, an expansion, an outside contraction, and an inside contraction respectively.

The algorithm flow for the maximization of the objective function is shown in Figure 3. The steps involved can be summarised as follows:

1. The objective function is evaluated at the initial working simplex (A_0, A_1, A_2).
2. The subsequent steps are repeated until the maximum value of the objective is obtained:
   - Calculate and compare objective values, $k(A_0)$, $k(A_1)$, $k(A_2)$.
   - If the maximum value of the objective is changed, transform the working simplex.
   - Terminate when the working simplex and objective are sufficiently small compared to the last simplex and objective value.
   - Return the best solution.

Simplex transformation involves:

1. Re-ordering solutions in increasing order of objective value.
2. Calculating the centroid of the two best points.
3. Computing the new working simplex by replacing the solution, with a minimum value of the objective, with a better solution either by:
   - reflection (A_2')
   - expansion (A_2^*) or
   - contraction (inside (A_2^{ic}) or outside (A_2^{oc}))

   with respect to the best point.

If 3 succeeds, the accepted solution becomes part of a working simplex. If 3 fails, then the simplex is shrunk towards the best solution. When a shrink is performed, all the solutions are thrown away and a new working simplex is developed.

III. Optimization in COMSOL

Problem formulation

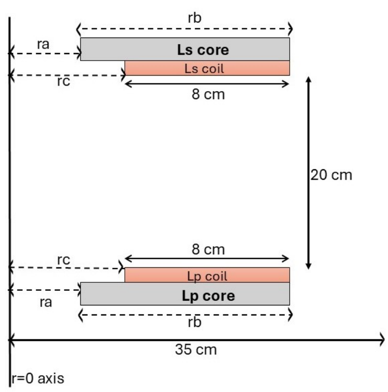


Figure 2: Problem formulation

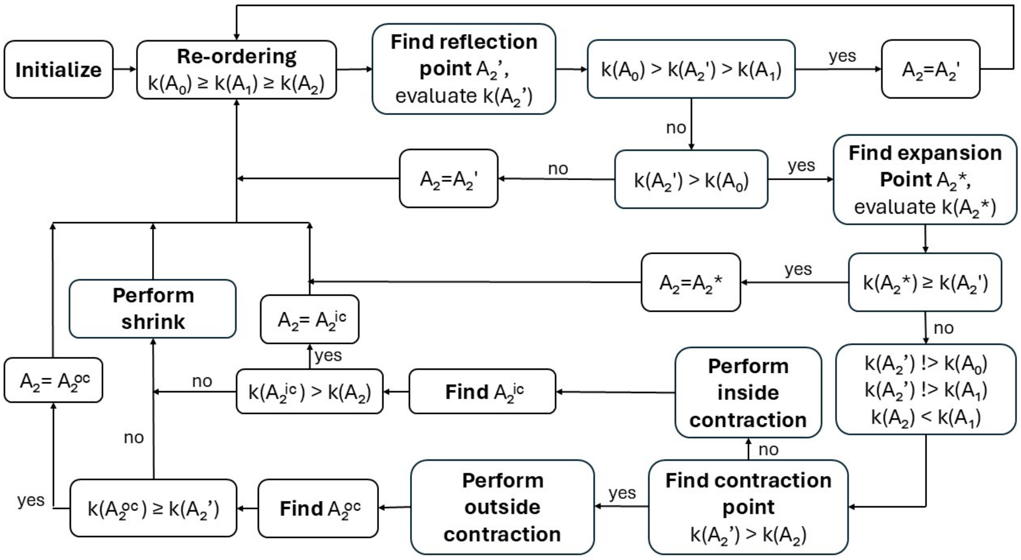


Figure 3: NM Simplex Algorithm flow for function maximization

For the equivalent circuit in Figure 1(b), L_p and L_s inductors are designed for maximum magnetic coupling/inductance. When L_p and L_s are symmetrical, the 2D axisymmetric problem is set as shown in Figure 2.

The L_p and L_s coils are planar and spiral. Each has 20 turns and a fixed width of 8cm (considering a 0.4cm litz wire diameter). For DC link voltages, V_i=V_o=400V (Figure 1(a)), V_1=510V at 85kHz is applied to the transmitting coil (Figure 1(b)). The core height is fixed at 1cm and the optimization variables can take values in the following ranges (also shown in Figure 3):

Core inner radius:	0 cm $\leq$ ra $\leq$ 20cm
Core width:	1 cm $\leq$ rb $\leq$ 35cm
Coil inner radius:	0 cm $\leq$ rc $\leq$ 27cm
Constraint:	ra+rb $\leq$ 35cm

(Constraint is placed on the maximum diameter of the IPT circular pad and that is less than or equal to 70cm).

Simulation

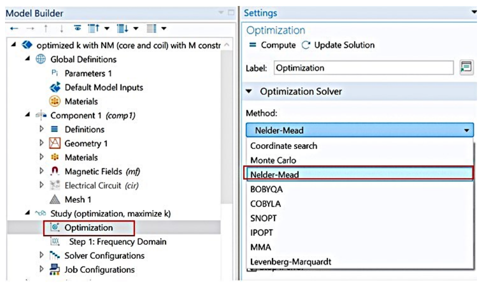
To solve the problem, the 'magnetic field' physics of COMSOL is used which solves for magnetic vector potential. The L_p and L_s coils are homogenized considering a litz wire that is applied at higher frequencies and can have a constant current density throughout its cross-section to overcome losses due to the skin effect. A frequency domain study is utilised to simulate the problem at 85kHz. In the optimization study step 'Nelder Mead' is selected from the options listed under 'Methods' in Figure 4(a). The objective function 'k' is described as an expression and the optimization type 'Maximization' is selected, as shown in Figure 4(b). The variables are defined with upper and lower limits and a constraint is placed on the maximum diameter of the IPT pads, as seen in Figure 4(c).

Results

The optimization process iterates until the maximum value of the objective function 'k' is obtained, as shown in Figure 5(a). The data points are highlighted by circles on the respective lines in Figure 5(a). The crowding of data points at the end of each line graph demonstrates that the difference in function value is very small, because the solution converges and the difference between solution points is small which leads to routine termination. The solution is:

$$(ra, rb, rc)=(4.1cm, 30.9cm, 25cm)$$

yielding the objective function value, k=0.28. The resulting geometry and its flux plot are shown in Figure 5(b).



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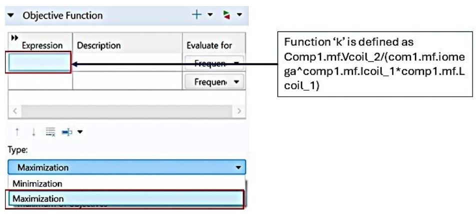

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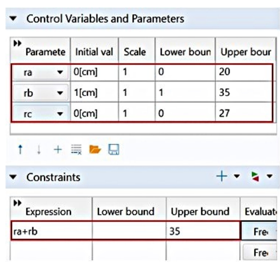
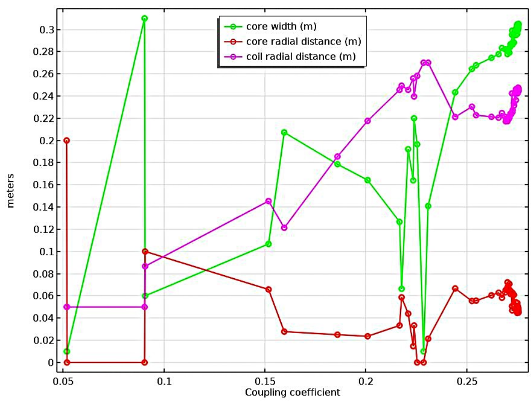


(c)

Figure 4: Simulation setting



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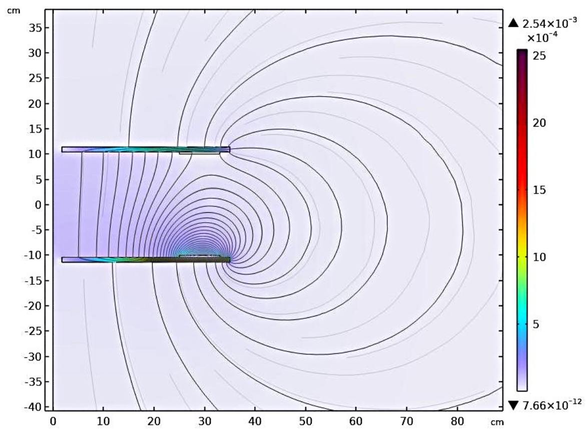


(b)

Figure 5

Validation through parametric sweep study

A parametric sweep study is performed on the coil inner radius, in COMSOL. This is done with the core and without the core. The resulting 'k' is shown in Figure 6 for both cases.

Without a core, 'k' increases as both L_p and L_s coils move away from the central axis. The higher the inner radius of the coil, the higher the 'k', and maximum 'k' is reached when this radius is equal to 29cm (see the black point in Figure 6). With a core, 'k' is always higher than without a core. Maximum 'k' is achieved at a lower value of coil inner radius, that is 25cm (see the red point in Figure 6).

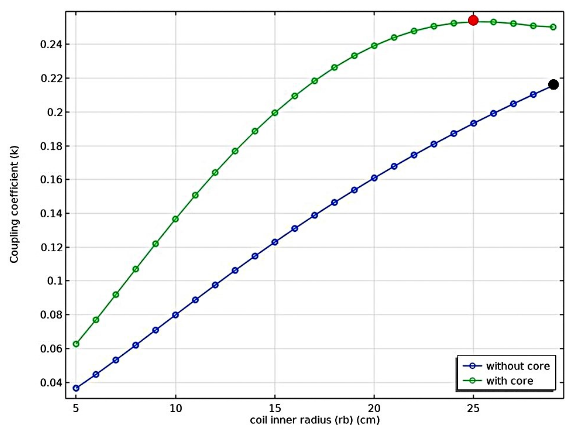


Figure 6

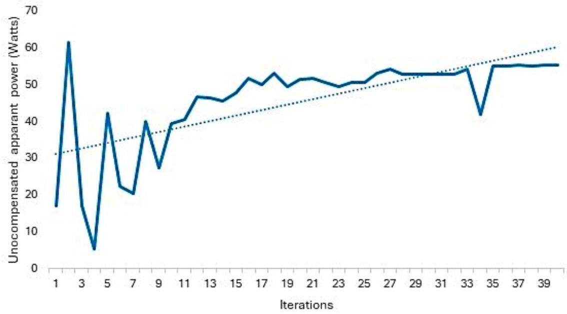
The optimum geometry of Figure 5(b) also shows the coil's inner radius of 25cm as the optimum value where 'k' is the maximum. This further validates the solution given by NM simplex in the last subsection.

IV. Discussion

The optimization study presented in this paper proves that COMSOL NM simplex provides a solution that can maximize certain magnetic parameters in electromagnetic systems. The method described to obtain higher magnetic coupling between electrically isolated coils can have significant importance in the design of IPT systems. For IPT coils, higher 'k' leads to a higher value of mutual flux. This in turn leads to a higher amount of power transferred, which is evident from eq. (1). The results can be further verified by measuring uncompensated apparent power (P_u) at the L_s terminals in each iteration of NM simplex. P_u is given by:

$$P_u = V_{oc} I_{sc} \tag{3}$$

V_{oc} is L_s open-circuit voltage and I_{sc} is L_s short-circuit current. The value that P_u takes in each iteration of NM simplex is shown in Figure 7(a).



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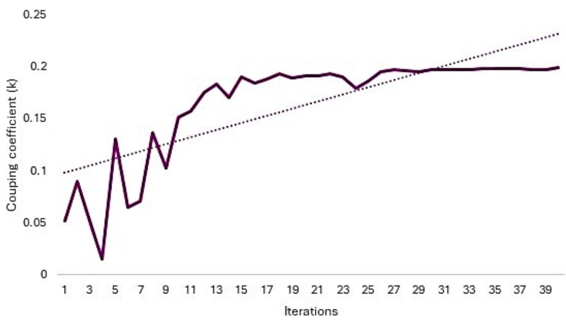

(b)

Figure 7: (a) Pu in NM iterations (b) k in NM iterations

A higher 'k' leads to a higher P_u. P_u shows the same trend as 'k' (illustrated by a dotted line in graphs 7(a) and (b)). Eventually, a geometry that gives a higher 'k' can transfer more power to the load which is the ultimate requirement of the IPT system for wireless charging of EVs.

V. Conclusion

In this paper,the COMSOL NM optimization routine was used for geometry design with maximum function value. IPT coils were designed with "magnetic field" physics and optimum geometry parameters were obtained from this optimization routine. The parameter to maximize was the magnetic coupling between the coils. NM in COMSOL returned geometry parameters that maximized the coupling in a few simple iterations. The resulting geometry from COMSOL NM optimization was validated by a separate parametric sweep study. This study gave the same highest coupling coefficient value which was obtained in the COMSOL NM optimization study step. At the 25cm inner radius of both coils, the coupling coefficient from both studies was found to be the same. This shows that the COMSOL NM optimization routine can be used to replace system evaluation at each possible set of variables. The latter is time-consuming and requires greater computational cost, while the former is straight forward and can give reliable solutions in less time.

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