High Fidelity TEA Theory Altered Compressible Navier-Stokes CFD Using COMSOL Equation-Based Modeling

The COMSOL CFD module includes High-Mach Number physics for simulation of supersonic flows and optional Spalart-Allmaras turbulence model. Consistent and inconsistent stabilization is entirely disabled and replaced with physics-based expressions based on Truncation Error Annihilation (TEA) theory. The altered equations are applied to AIAA High-Fidelity benchmark problems Sajben Diffuser and Smooth Bump. Results are validated and variations in mesh, discretization order, and solution methods investigated.

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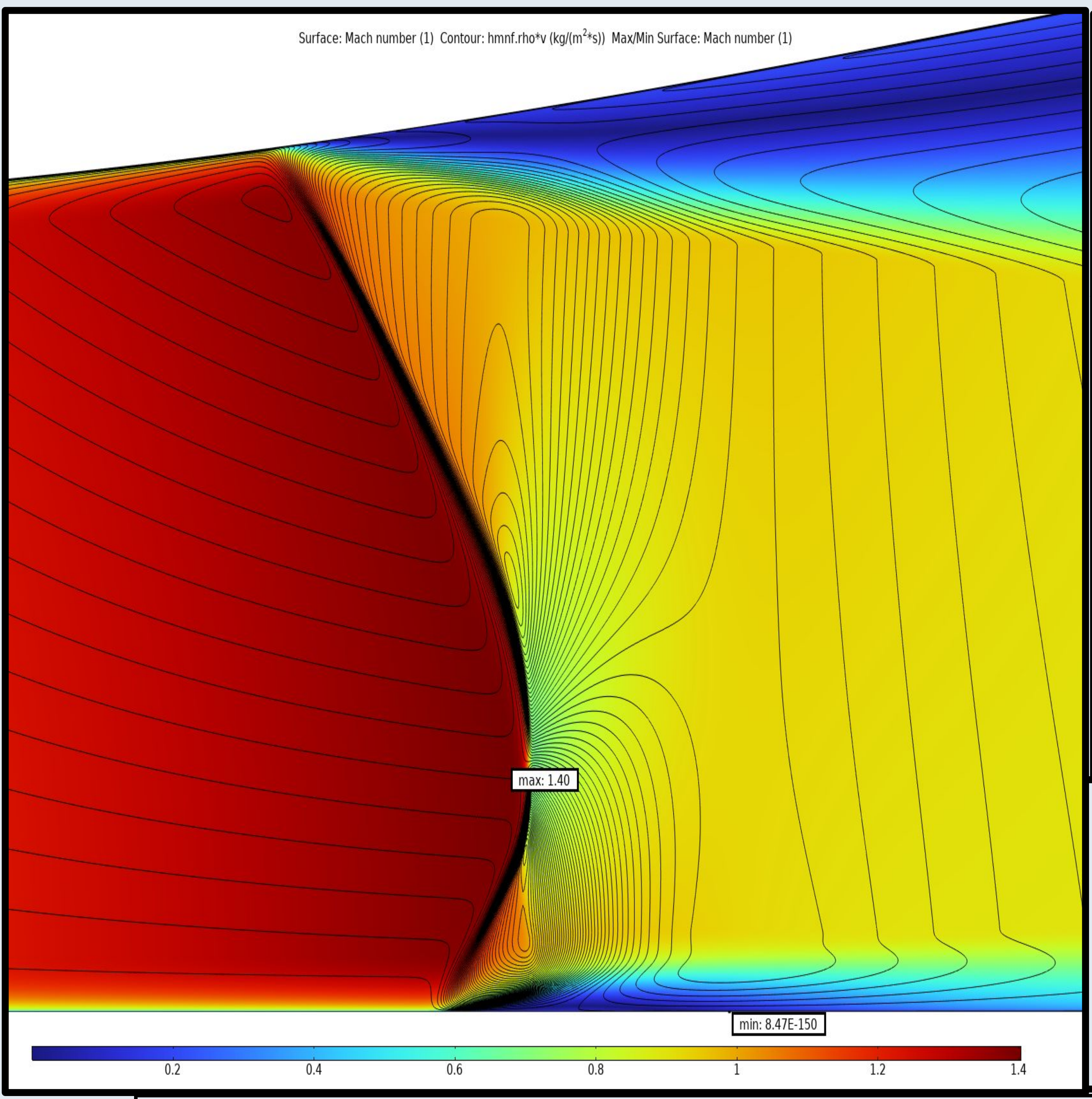
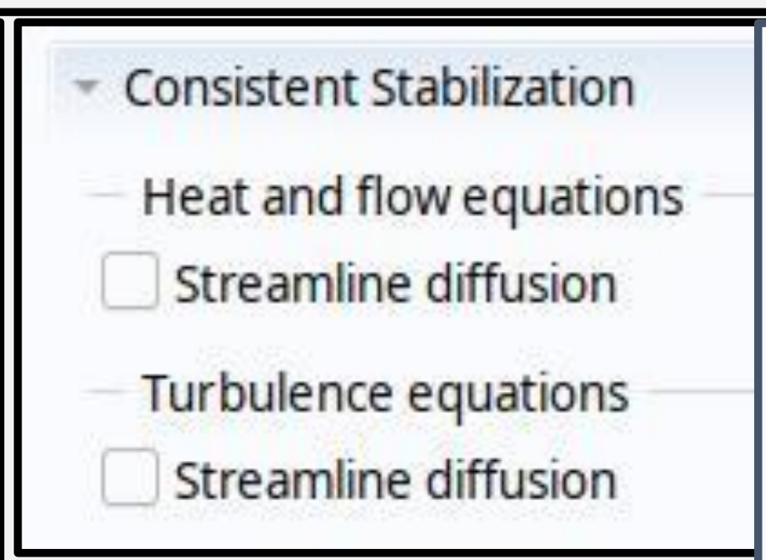


Truncation Error Annihilation (TEA) Theory and Implementation in COMSOL

Essential TEA theory equation to alter compressible Navier Stokes

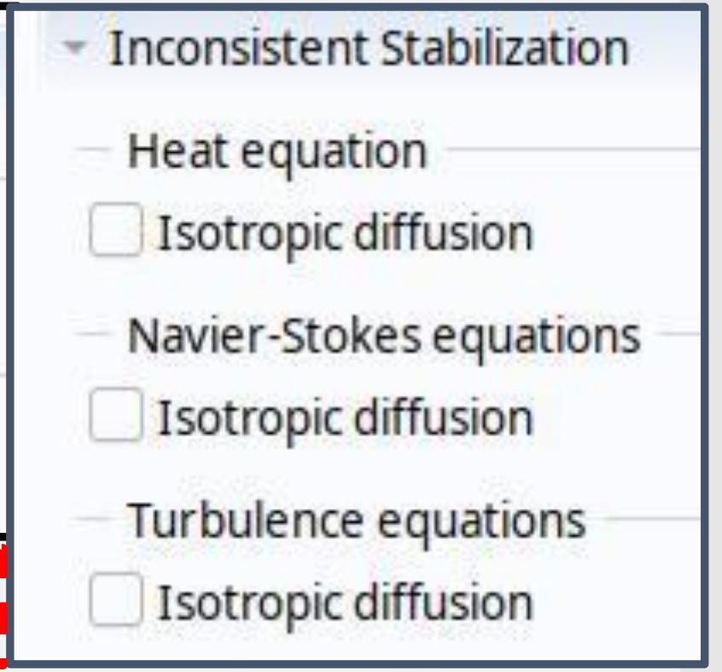
$$\cdots - \frac{Re^*\left(\frac{h_e}{m}\right)^2}{12}\left[u^2\frac{\partial^2 q}{\partial x_1^2}+v^2\frac{\partial^2 q}{\partial x_2^2}\right]=0+O(h^4).$$



▾ Consistent Stabilization

Heat and flow equations
☐ Streamline diffusion

Turbulence equations
☐ Streamline diffusion



▾ Inconsistent Stabilization

Heat equation
☐ Isotropic diffusion

Navier-Stokes equations
☐ Isotropic diffusion

Turbulence equations
☐ Isotropic diffusion

stabilization off

Equations are coupled with a sequence of altered terms in weak form.

u
```
-(h_2d/order)^2/12/mu_tot*
(test(ux)*u^2*ux+test(uy)*v^2*uy)
```

v
```
-(h_2d/order)^2/12/mu_tot*
(test(vx)*u^2*vx+test(vy)*v^2*vy)
```

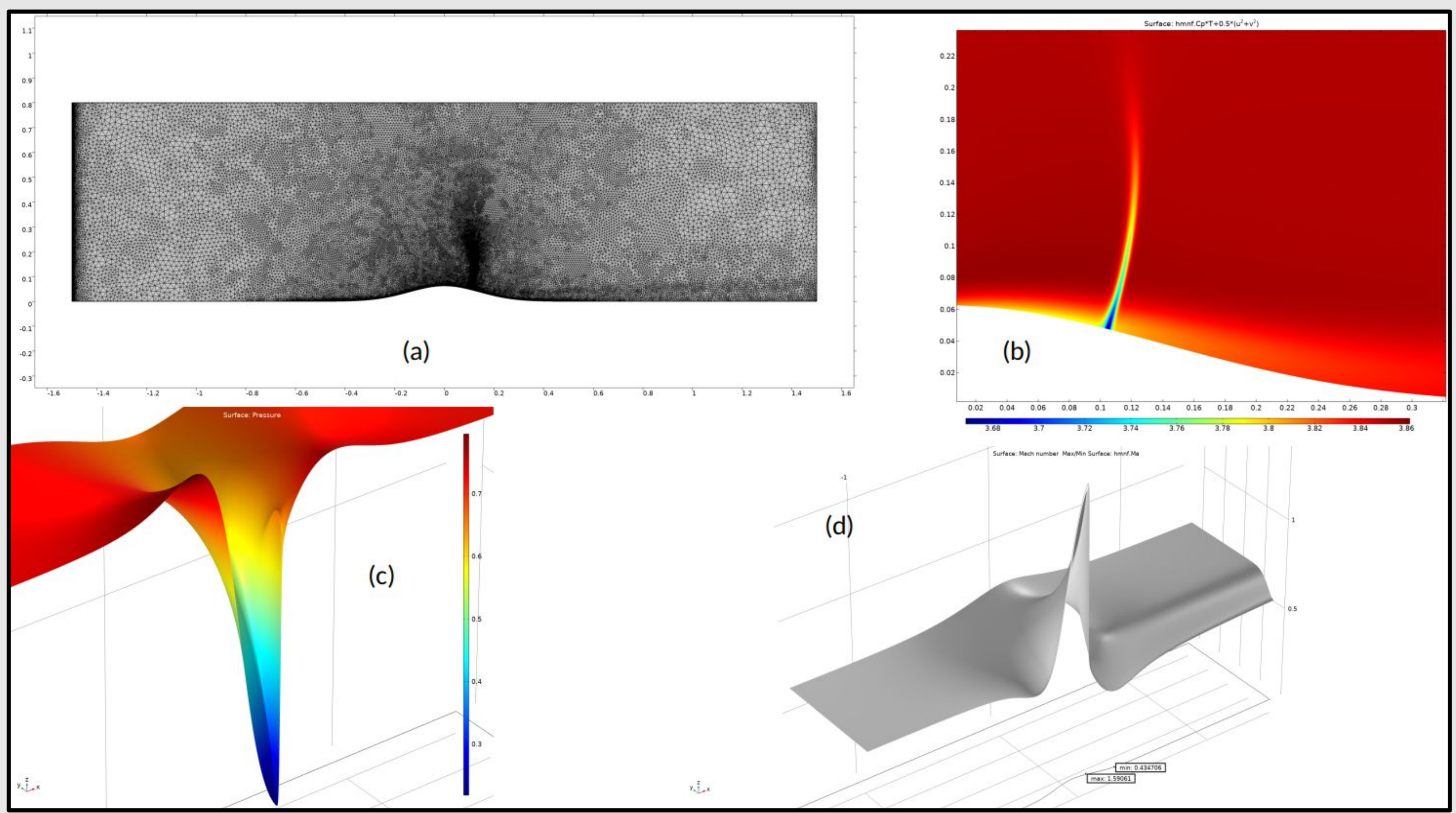
T
```
-(h_2d/order)^2/12/k_tot*
(test(Tx)*u^2*Tx+test(Ty)*v^2*Ty)
```

see full paper

special treatment for continuity equation

μ_T
```
-(h_2d/order)^2/12*hmnf.sigmanu/nu_tot*
(test(nutildex)*u^2*nutildex
+test(nutildey)*v^2*nutildey)
```

p
```
+h_2d/order/tau_p*(test(px)*px+test(py)*py)
```
```
-(h_2d/order)^2*tau_p/L_p^2*
(test(px)*u^2*px+test(py)*v^2*py)
```



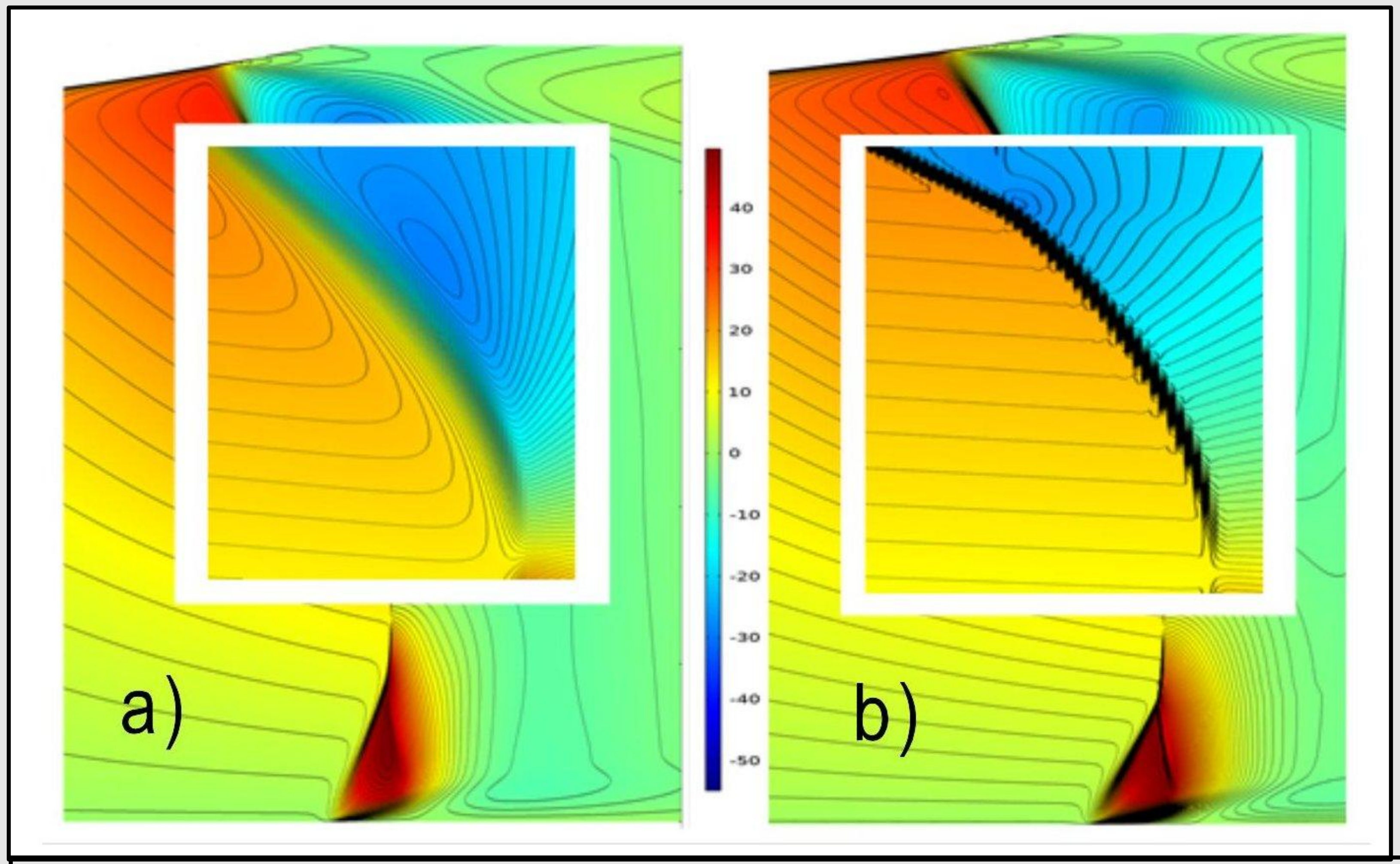
TEA *O*(h^4) transonic smooth-bump collage of interesting results (a) cubic basis adaptive mesh (b) total enthalpy distribution, (c) pressure surface height-enabled, and (d) Mach number distribution height-enabled.

Transonic Smooth-Bump Validation

One of several plots in the full paper, a collage: (a) the final adaptive mesh for the cubic basis; in addition to the high mesh density about the shock, the adaptive mesh algorithm also refined the inlet edge region, and a significant portion of the bump edge; total number of cubic elements for this case is 134776. (b) the total enthalpy distribution; theoretical constant value is a constant (H_o=3.843); nearly all the distribution is very near this quantity; however, in the shock vicinity there is a significant dip down to H_o ~ 3.68. (c) pressure surface, detailing additional plateau developed within the shock wall (d) Mach number, gray-shade surface detailing zero gradient in the normal direction for all the edges, including the bump edge; maximum Mach number slightly inside the domain near the bump edge.

Sajben Diffuser Validation

One of several figures in the full paper, this figure shows 2D plots of detail about the shock region. A comparison plot of Truncation Error Annihilation (TEA) theory application and COMSOL consistent stabilization (CCS) zoom surface. Contour lines of transverse (v) velocity are also added for increased definition of the shock boundary-layer interaction behavior. An additional zoom overlay is included of a critical region near the upper wall boundary layer interaction. The left side, a) TEA, demonstrating smooth. monotone, accurate solution, whereas the right side, b) CCS demonstrates an oscillatory, unstable, and inaccurate behavior.



TEA Theory *O*(h^4) Sajben diffuser comparison, transverse velocity (v), shock-centric zoom with super-zoom snippet imposed:
a) TEA theory *O*(h^4), and b) COMSOL consistent stabilization *O*(h^2).

REFERENCES

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