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A numerical model for the unified analysis of soil sedimentation-consolidation phenomena

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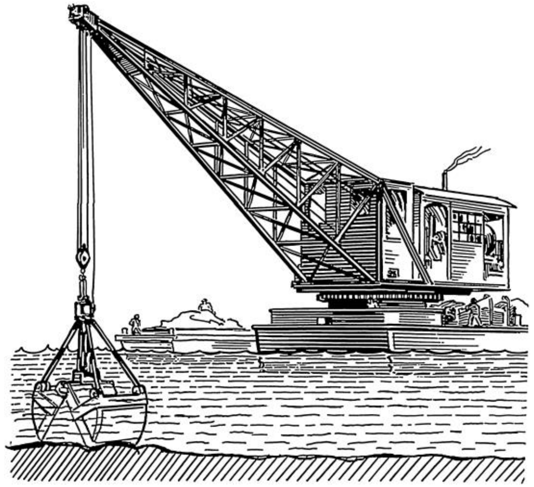
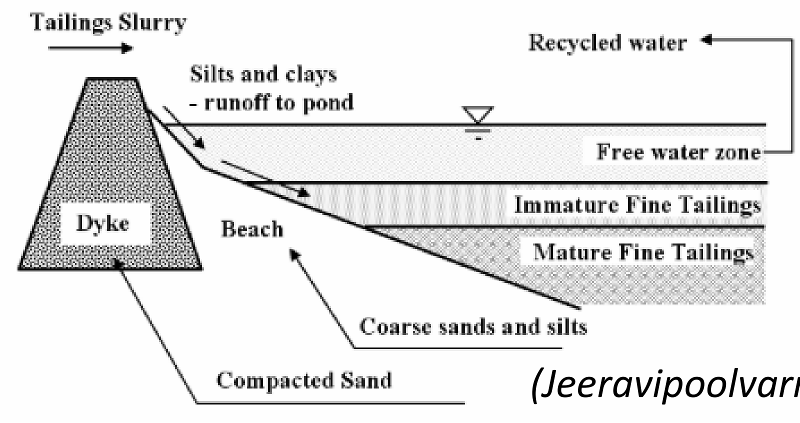
Relevant engineering applications

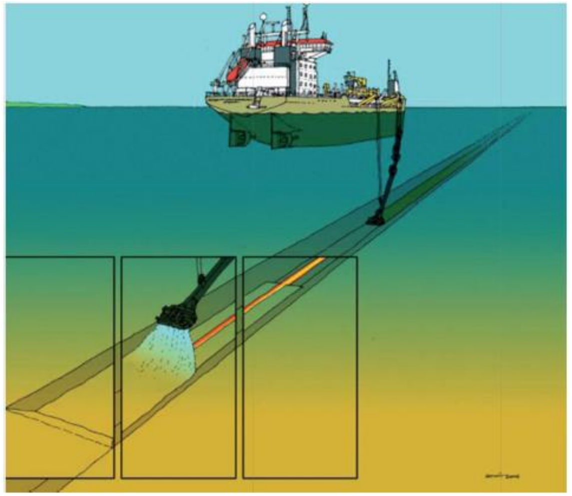


Dredging

Oil sand tailings



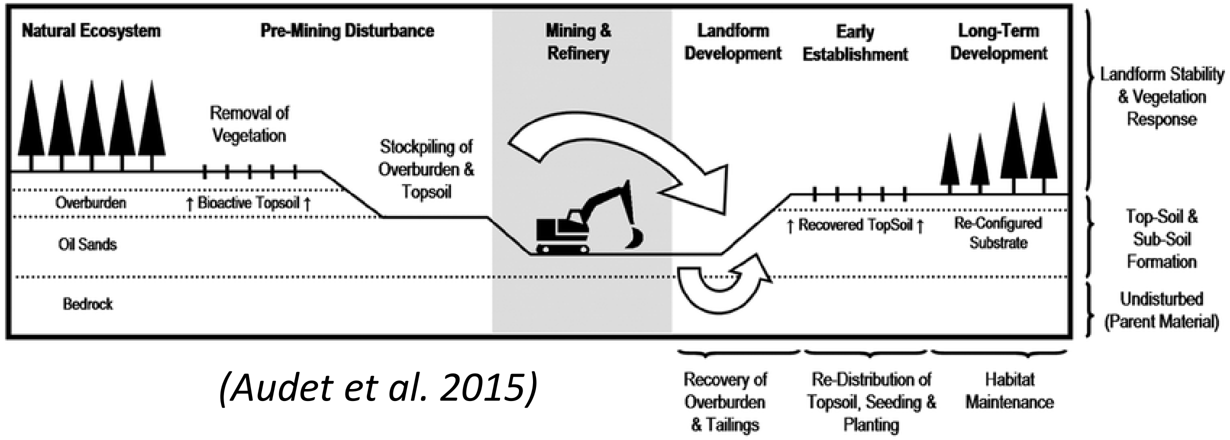
(Jeeravipoolvarn, 2010)



Submarine pipeline trench backfilling

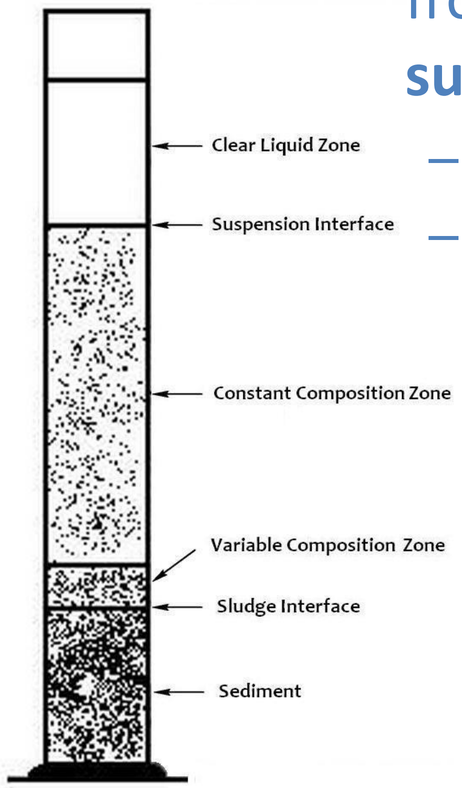
(Stijn Biemans 2012)

Land reclamation



(Audet et al. 2015)

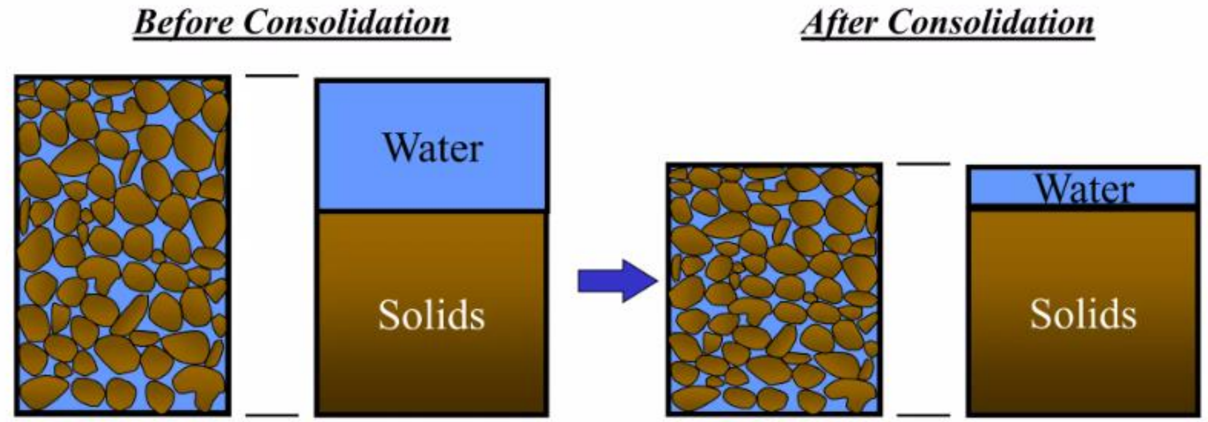
(1) Sedimentation & (2) Consolidation



- Deposition of solid material from a fluid from a state of **suspension**
  - «Fluid» state
  - Absence of interparticle force chains

- Gradual volume reduction in saturated **soil** due to pore fluid drainage
  - «Solid» state
  - Formed sediment network structure able to carry its own weight

Irfan (2016)



(1) Kynch's theory of sedimentation

Hp:

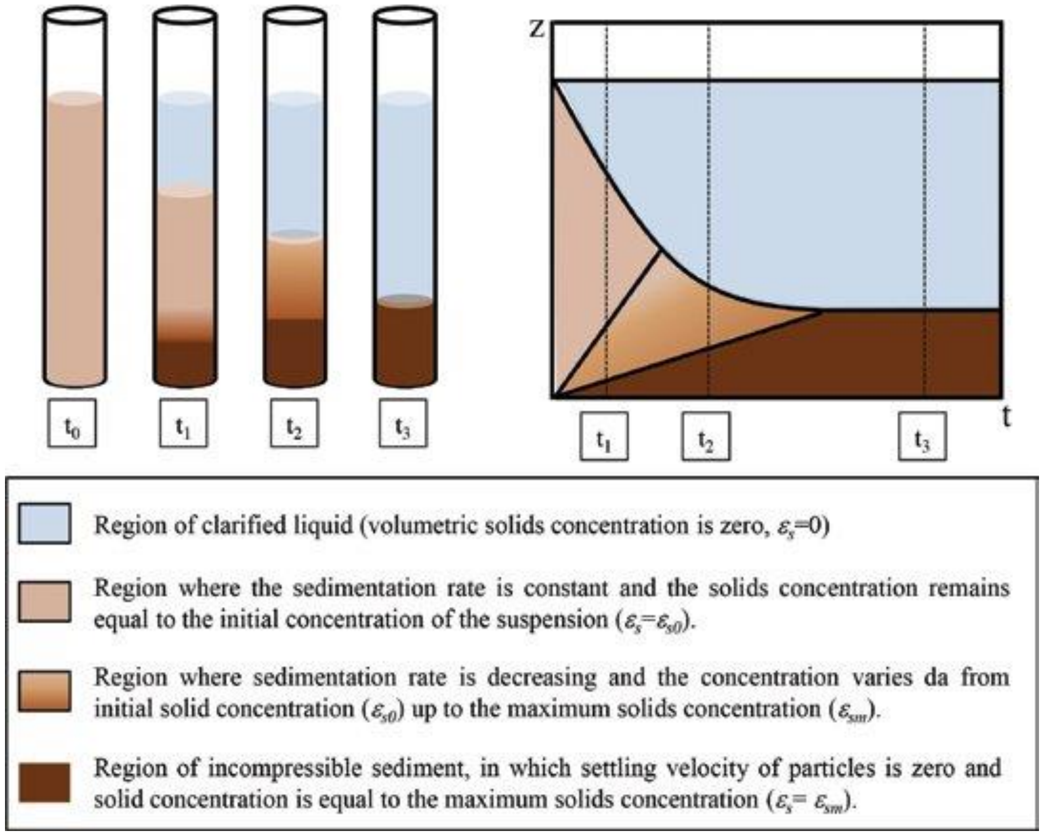
- $v_s = v_s(c)$
- Continuity of solid and fluid phases

$$V(c)\frac{\partial c}{\partial x} + \frac{\partial c}{\partial t} = 0$$

$$V(c) = v_s + c\frac{dv_s}{dc}$$

Hindered settling equation
(Kynch 1951)

- Eulerian coordinate formulation
- *c* = solid mass per unit volume

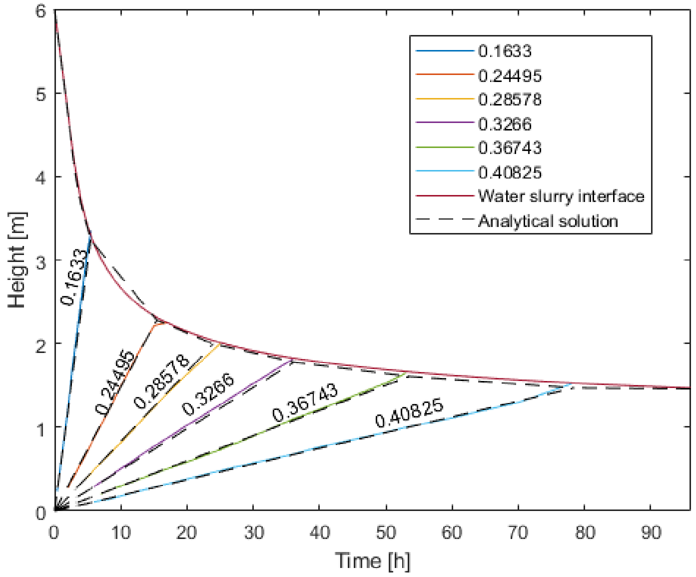


Kynch’s theory numerical implementation

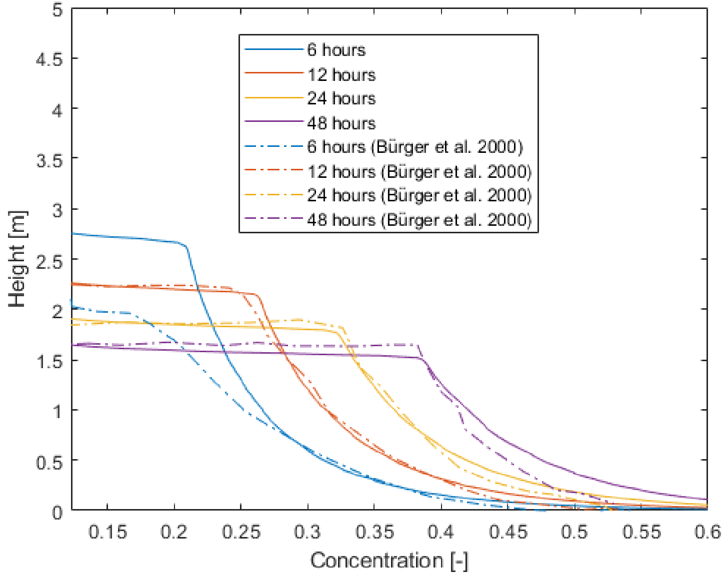
- Comsol implementation in Lagrangian coordinate
- Comparison with analytical and numerical solutions from literature

$$V_z(e)\frac{\partial e}{\partial z}+\frac{\partial e}{\partial t}=0$$

$$V_z(e)=\left(\frac{\gamma_s}{\gamma_f}-1\right)\frac{d}{de}\left[\frac{k}{1+e}\right]$$



Comparison with analytical solution (characteristics) Evolution of solid-liquid interface position & constant concentration lines



Comparison with numerical solution of Bürger et al. (2000)

(2) Large strain 1D consolidation theory

Gibson et al. (1967)

- Continuity equation for solid and fluid phases
- Darcy's law

Lagrangian coordinate

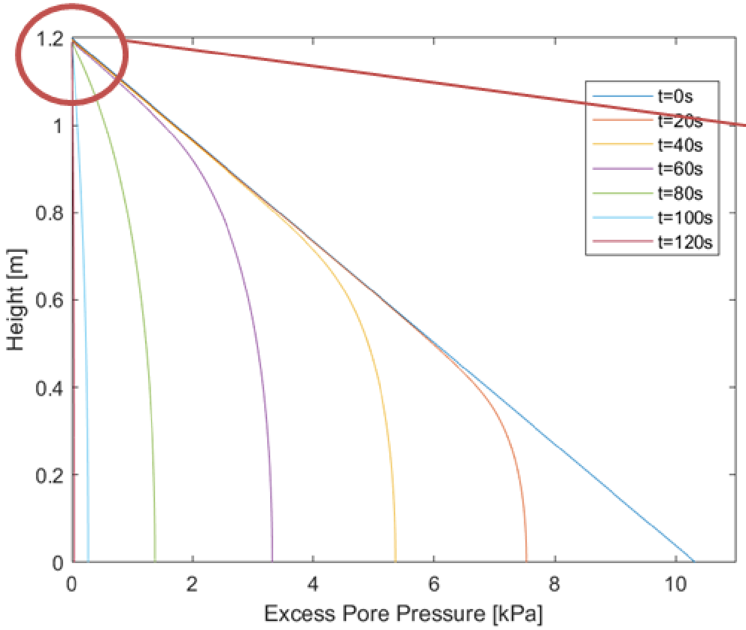
$$z(x) = \int_0^x \frac{1}{1+e} dx$$

$$k = k(e)$$

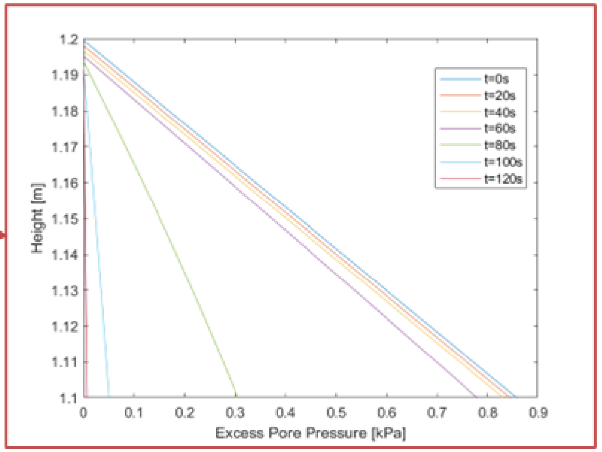
$$\sigma' = \sigma'(e)$$

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f}\frac{1}{1+e}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] \mp (\gamma_s - \gamma_f)\frac{d}{de}\left[\frac{k}{\gamma_f}\frac{1}{1+e}\right]\frac{\partial e}{\partial z}$$

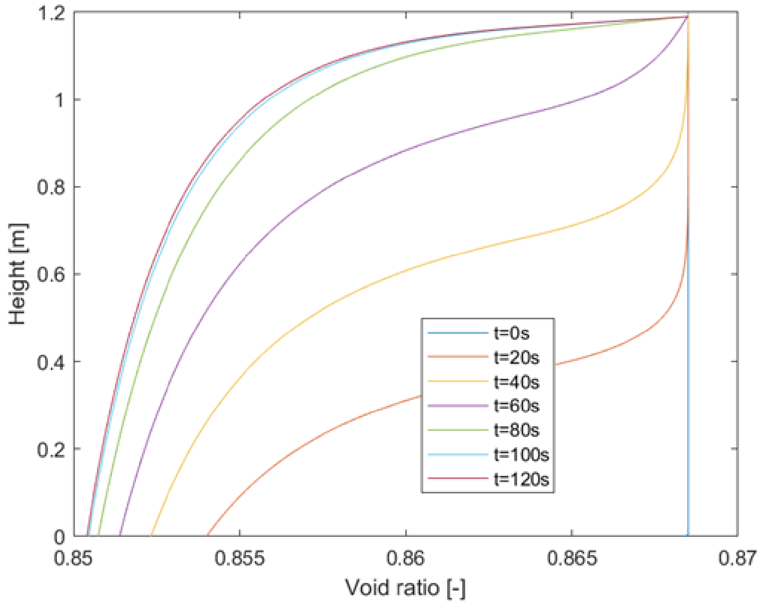
Large strain consolidation numerical implementation



Excess pore pressure isochrones
Eulerian coordinate



Layer compression over time



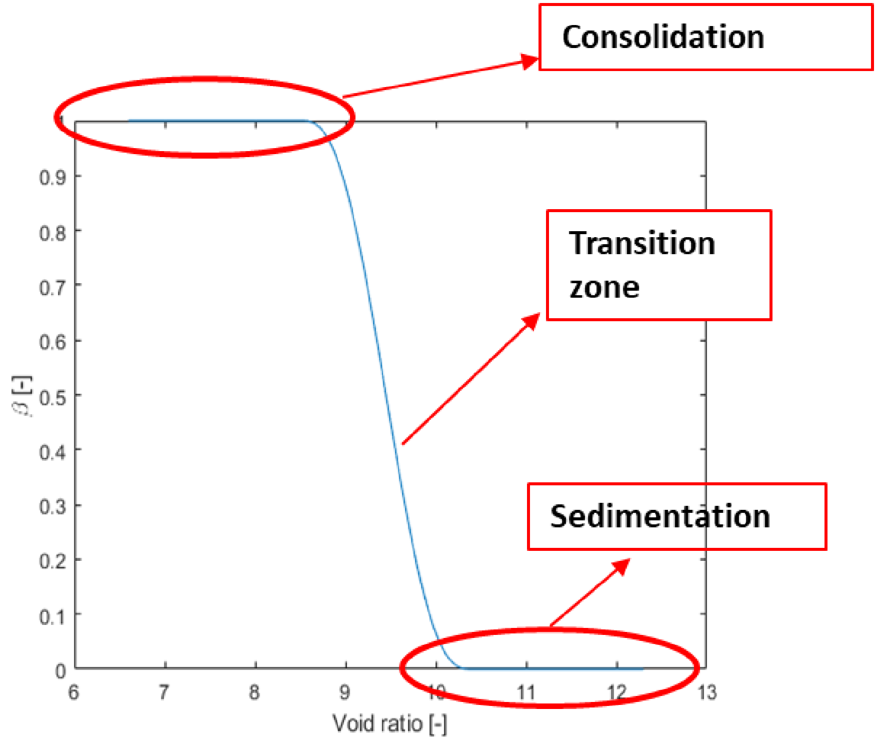
Void ratio isochrones
Eulerian coordinate

Sedimentation+consolidation:

Interaction coefficient

More general form of effective stress principle via interaction coefficient $\beta(e)$

$$\sigma' = \beta(e)(\sigma - u)$$



- Used to model transition between sedimentation and consolidation
- Defined via step function

$$\beta(e) = \begin{cases} 1 & e \le e_s \\ a_5 e^5 + a_4 e^4 + a_3 e^3 + a_2 e^2 + a_1 e + a_0 & e_s < e < e_m \\ 0 & e \ge e_m \end{cases}$$

Step Function in COMSOL Multiphysics

Sedimentation+consolidation

Governing equation
(Pane & Schiffman 1985)

- $$\frac{\partial e}{\partial t} = \mp\left(\frac{\gamma_s}{\gamma_f} - 1\right)\frac{d}{de}\left(\frac{k}{1+e}\right)\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\beta(e)\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\frac{d\beta(e)}{de}\sigma'\frac{\partial e}{\partial z}\right]$$

Sedimentation+consolidation

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$\beta(e) = 0$

$e \geq e_m$

Kynch's equation

$$\frac{\partial e}{\partial t} \pm V_z(e)\frac{\partial e}{\partial z} = 0$$

Sedimentation+consolidation

Governing equation
(Pane & Schiffman 1985)

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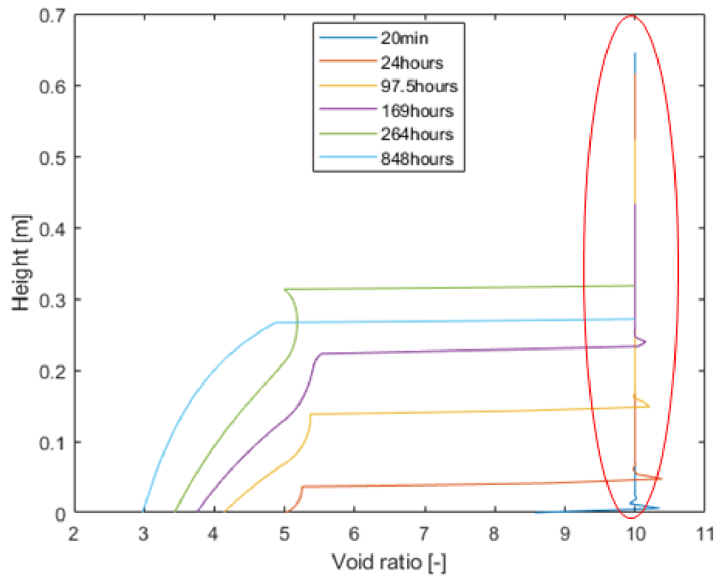

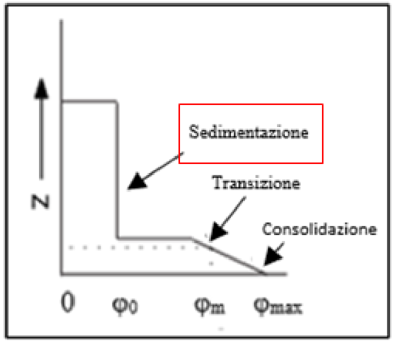

(da Dankers 2006, modificato)

Sedimentation+consolidation

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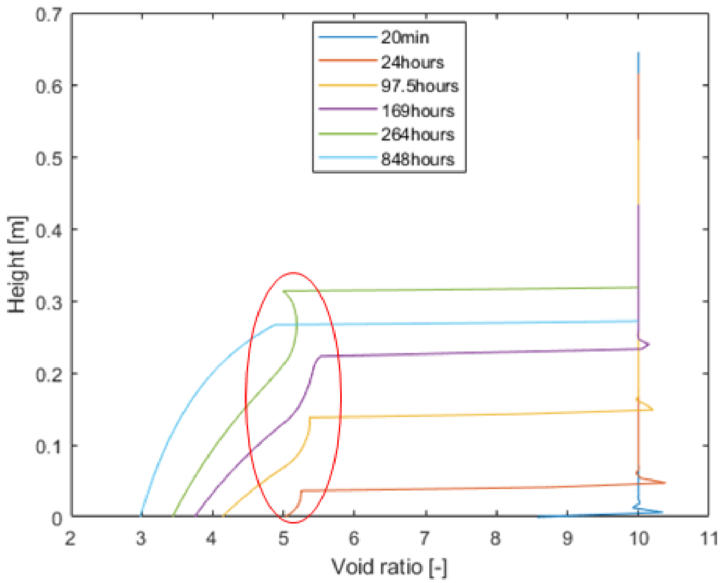
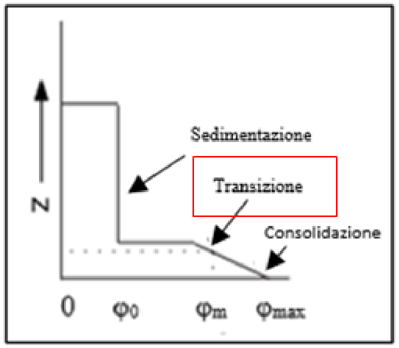
$\beta(e)$

$e_s < e < e_m$





(da Dankers 2006, modificato)

Sedimentation+consolidation

Governing equation
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β(e) =1

$e \leq e_s$

Consolidation
Gibson equation

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f}\frac{1}{1+e}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] \mp (\gamma_s - \gamma_f)\frac{d}{de}\left[\frac{k}{\gamma_f}\frac{1}{1+e}\right]\frac{\partial e}{\partial z}$$

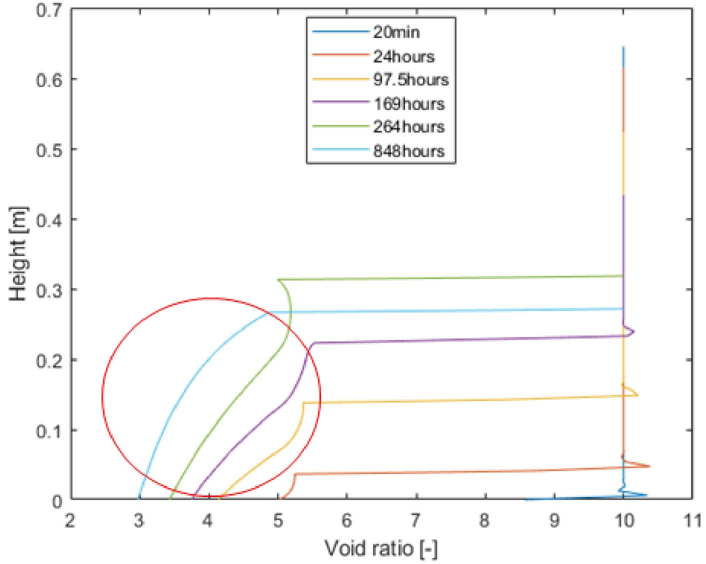
Sedimentation+consolidation

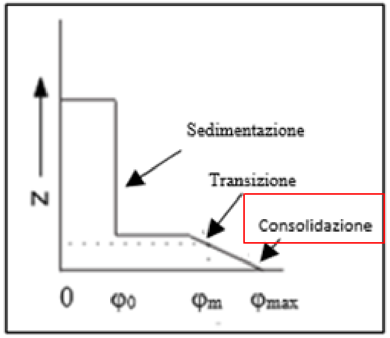
Governing equation
(Pane & Schiffman 1985)

- $$\frac{\partial e}{\partial t} = \mp\left(\frac{\gamma_s}{\gamma_f} - 1\right)\frac{d}{de}\left(\frac{k}{1+e}\right)\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\beta(e)\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\frac{d\beta(e)}{de}\sigma'\frac{\partial e}{\partial z}\right]$$

$\beta(e) = 1$

$e \leq e_s$





(da Dankers 2006, modificato)

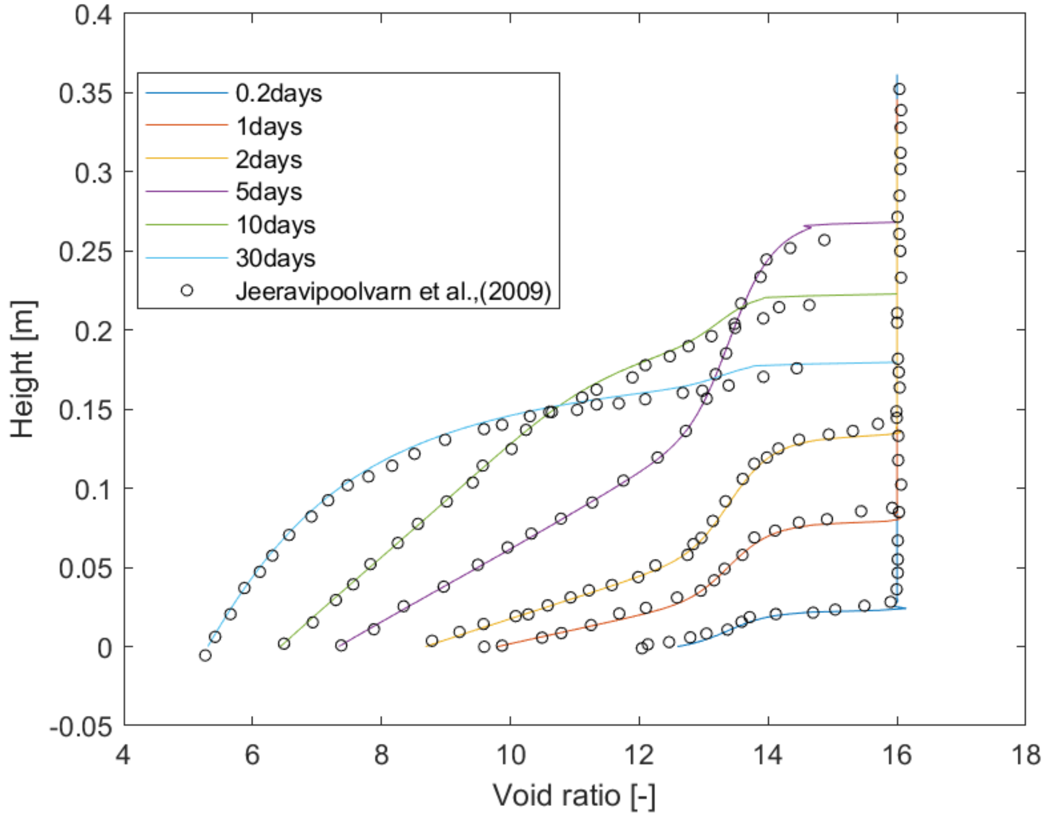
Validation of FE model

- Validation against numerical solution (Jeeravipoolvarn 2009)

$$k = Ce^D$$

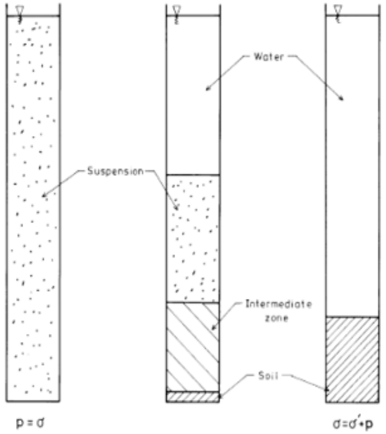
$$e = A(\sigma')^B$$

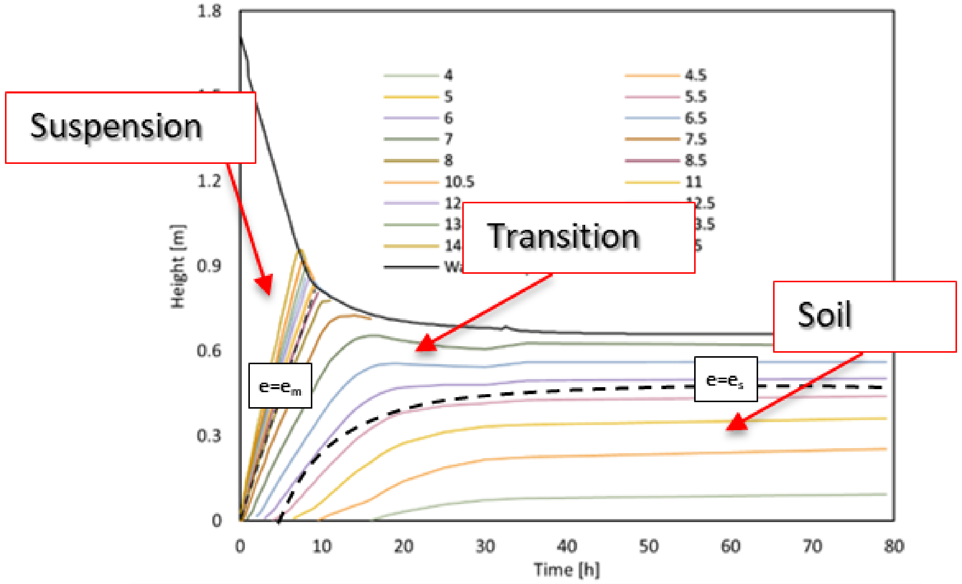
$$\beta = \begin{cases} \left(\dfrac{1}{E + Fe^G}\right), & \beta > \beta_t \\ \beta_t, & \beta \leq \beta_t \end{cases}$$

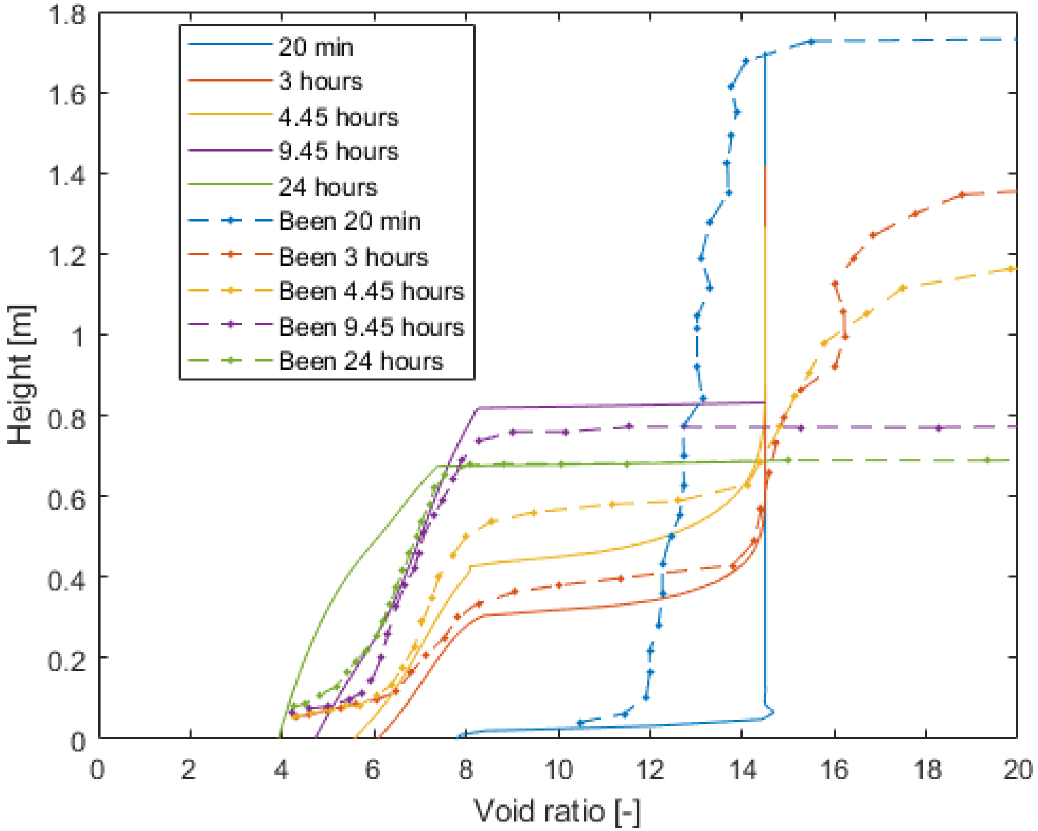


Validation against experimental data

Been 1980



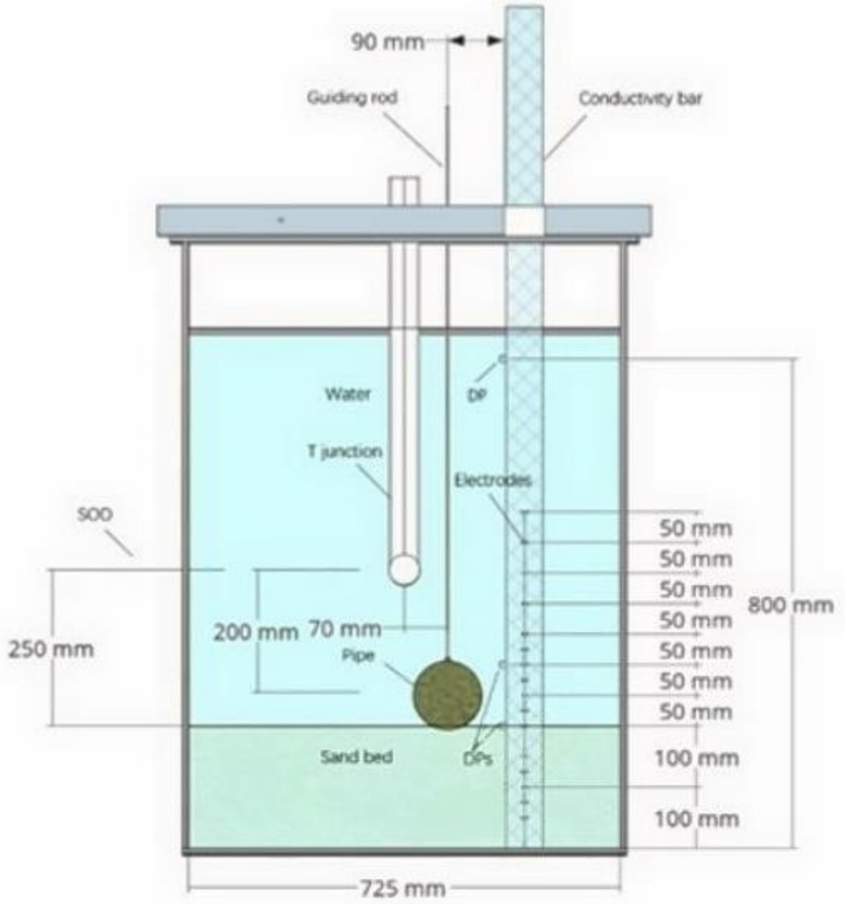


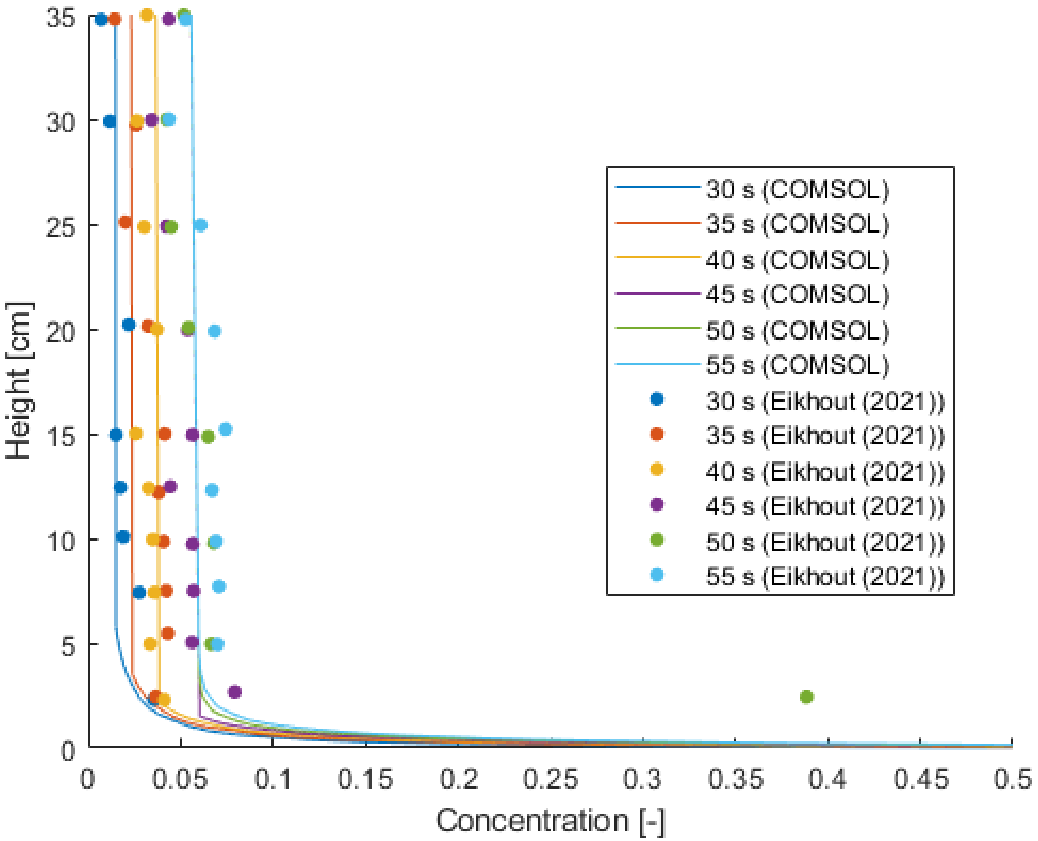


Experiment 7

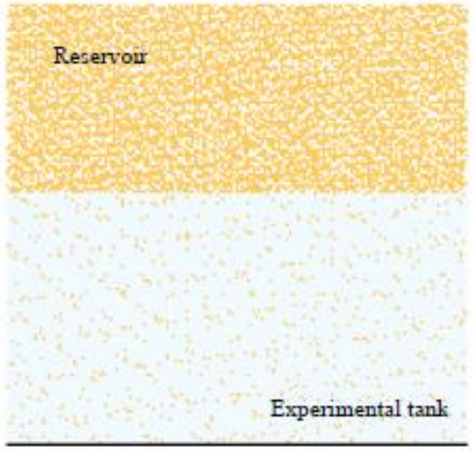
Validation against experimental data

Eikhout 2021





Test 1



Dummy resevoir representation

Conclusions

- The numerical model can simulate
  - ✓ Large-strain consolidation
  - ✓ Sedimentation
  - ✓ Sedimentation-consolidation
- Model validation against experimental data
  - ✓ Simulation of sedimentation-consolidation processes involving clayey material
    - Application to land reclamation problems
  - ✓ Simulation of sedimentation due to inflow of sand suspension
    - Application to underwater trench backfilling and pipeline-soil interaction problems

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