Numerically Generated g-functions for Ground Coupled Heat Pump Applications

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Abstract: In most ground-coupled heat pump systems, Borehole Heat Exchangers (BHE) represent the typical engineering solution for utilizing renewable energy from the ground. The design of a complex BHE field is a challenging task, due the inherent transient nature of the thermal interaction between the heat exchangers and the surrounding soil. A computation effective method for solving the 3D transient conduction equation describing the ground response to a variable heat load profile is the temporal superposition of pre-calculated temperature response factors or g-functions. In this study Comsol heat conduction models have been developed to calculate g-function values for a borehole field with 64 boreholes. The aim of the investigation is to get an insight on the numerical generation of temperature transfer functions and to some extent provide new information on the Finite Line Source method for analytically generated g-functions as well as on those existing behind existing design software such as EED. The results generally showed a good agreement in lower time ranges. Further in time, the Comsol model revealed to be influenced either by the domain dimensions or the simulation end time.

Keywords: g-function, borehole heat exchanger, long term performance, FLS, EED, numerical, ground coupled heat pumps.

1. Introduction

The Ground Coupled Heat Pump (GCHP) systems have been studied actively during the last decades, being accepted as one of the most sustainable technology with very high efficiency and low environmental impact.

GCHP systems consist of a set of heat exchanger pipes buried in the ground and coupled to a heat pump. In most cases vertical ground heat exchangers where a fluid is circulated inside a system of pipes is inserted in a deep vertical borehole drilled in the soil. The ground is used as heat source and/or sink to provide space heating or cooling in commercial and residential buildings. The number of boreholes, their length, as well as the distance between them is defined previous to construction based on the thermal response of the ground to the energy demand for very large scales of time. Medium to long term ground thermal behavior analyses must take into consideration the time varying heat loads from/to the ground due to the building requirements, the borehole field geometry, and the ground thermo-physical properties.

The thermal response of the ground is evaluated using either analytical or numerical approaches. For very simple BHE geometries and under proper assumptions, some analytical solutions (temperature response factors) are available (Ingersoll et al., 1954). When many BHEs are present, simple analytical solutions cannot be applied for long term analyses and dedicated g-functions have to be calculated. This calculation, which must include a time range up to decades, can be performed numerically, by solving the 3D transient heat conduction differential equation.

The first and most famous extensive research devoted to the calculation of proper temperature response factors (or g-functions) is the one by (Eskilson, 1987). Another approach is again a hybrid method where a single BHE solution, the Finite Line Source Solution (FLS) is either superposed in time and in space, to describe the ground response of any BHE geometry. FLS generated g-functions are based on a constant heat flux at BHE periphery as boundary condition and, in general, they fit well the reference functions by (Eskilson, 1987).

The purpose of this paper is to numerically calculate, for one borehole field geometry, g-function values by means of Comsol simulations solving the 3D problem with the transient heat conduction equation. This, as first step to get an insight on the numerically generated g-functions and also try providing new information on reliability of FLS generated ones as well as on a function obtained from the design software EED, especially with reference to the “asymptotic” part of the g-function profiles and with reference to the boundary condition effects on temperature response factor shape.

2. Theoretical background

A vertical BHE field thermally interacts with the surrounding ground according to the laws of the three-dimensional transient heat conduction equation, provided groundwater circulation can be neglected. The complete conduction equation is seldom solved due to its intrinsic computational demand related to the huge dimensions of the calculation domain (up to thousands of cubic meters), to the great time horizon of the analysis (decades), to the short time step needed for detailed results (from hours to months) and to the time-varying boundary conditions (heat loads to the ground).

For the above reasons, a series of simple solutions (from the point of view of the geometry and of the boundary conditions) have been developed. Combined with proper superposition techniques, these solutions can be used to obtain a quasi three-dimensional solution describing the borehole field behaviour in real operating conditions (multi-annual hourly simulations), while assuring relatively short computational times.

Analytical approaches as the Infinite Line Source (ILS) theory introduced by Kelvin in 1882 and later cited by (Ingersoll et al., 1954), describe the BHE as an infinitely long linear heat source/sink. The ground thermal properties are assumed to be constant and the source is represented by constant heat transfer rate per unit length.

New contributions to the linear source theory have been provided in the two last decades. The Finite Linear Source (FLS) model, which better describes a real single BHE, was first studied also by (Eskilson, 1987), later by (Zeng et al., 2002) and recently by (Lamarche and Beauchamp, 2007). The latter is based on a mirroring technique and on a double integration of the Fourier equation which provides the ground borehole radial temperature (including the one at the borehole periphery, $r=r_{bhe}$) as the average along the depth z. The solution is expressed in terms of the complementary error function (*erfc*), as expressed in Eq. (1):

$$T_{ave}(r) - T_{gr,\infty} = \frac{\dot{Q}'}{2\pi k}\left[\int_{\beta}^{\sqrt{\beta^2+1}} \frac{erfc(\gamma z)}{\sqrt{z^2-\beta^2}}dz - D_A - \int_{\sqrt{\beta^2+1}}^{\sqrt{\beta^2+4}} \frac{erfc(\gamma z)}{\sqrt{z^2-\beta^2}}dz - D_B\right] \quad (1)$$

where β is the dimensionless (with respect to BHE depth H) radial distance, and γ is a function of the Fourier number based on the BHE depth, Fo_H, as expressed in Eq. 2.

$$\gamma = 0.5\left(\frac{\alpha\ t}{H^2}\right)^{-0.5} = 0.5 Fo_H^{-0.5} \quad (2)$$

In Eq. (1), D_A and D_B are functions of *erfc* but they are constants at given time t and depth H. The FLS solution can be numerically solved by a proper iterative technique, for example through the extended midpoint algorithm for improper integrals (Press et al., 1992). Of practical interest is the calculation of the time varying temperature excess at the borehole wall, $r = r_{bhe}$.

The superposition of single solutions can be made to simulate the response of different BHEs in a borehole field to a given thermal load, known as spatial superposition. Spatial superposition can be employed for developing the above mentioned g-functions that describe the thermal behaviour of a ground volume in contact to a given geometry BHE arrangement. The g-functions are expressed as in Eq. (3).

$$T_{ave}(r_{bhe}) - T_{gr,\infty} = \frac{\dot{Q}'_{ave}}{2\pi k} g(\ln(9Fo_H, r_{bhe}/H, B/H, borefield\ geometry) \quad (3)$$

In Eq. 3, $T_{ave}(r_{bhe})$, is the average borehole wall temperature, and Q'_{ave} is the average heat transfer rate per unit length for the whole borehole field.

Once a proper g-function is available for a given geometry, the linear properties of the conduction equation can be exploited for evaluating the effects of variable heat pulses (and not only a constant one) on the ground response and GCHP system performance. This technique is known as temporal superposition (Ingersoll et al., 1954), (Eskilson, 1987), (Yavuzturk and Spitler, 1999), (Bernier et al. 2004).

(Eskilson, 1987) and (Hellstrom and Sanner, 2001) have evaluated hundreds of g-*functions* using spatial and temporal superposition for a huge number of borehole field configurations but, unfortunately, they are just implemented in commercial software such as EED (Hellstrom and Sanner, 1994) they are still somewhat limited by the predefined configurations of straight boreholes. Using Comsol allows for more flexibility although the computational time increases significantly.

A perhaps better flexibility is achieved with the FLS model proposed by (Lamarche and Beauchamp, 2007) even for inclined boreholes (Lamarche, 2011), who demonstrated that their analytical solution, valid for imposed heat flux, can be applied for g-function generation by proper superposition. (Sheriff, 2007), (Bernier and Cauret 2009) and (Fossa, 2011a) confirmed that the FLS base solution (and even approximations of the FLS itself (Fossa, 2011b) can be successfully. These authors noticed a general good agreement between FLS generated g-functions and those from (Eskilson, 1987). On the other hand some differences (of the order of circa 10% for large borehole fields) have been found especially towards the asymptotic region of the g-function profile. These discrepancies could be attributed to the different way the g-functions have been calculated, even if some preliminary findings reported in (Fossa et al., 2009) seem not to confirm this explanation.

3. Generation of g-functions with Comsol

In the present paper, the choice has been to model a 8x8 borehole arrangement using Comsol (64 boreholes placed in a rectangular configuration). The thermal properties of the material in the simulated domain are listed in Table 1. This domain has been created in Comsol by subtracting all single borehole heat exchangers volume from the surrounding ground.

Table 1. Thermal properties in modeled domain

Cp [J/kg K]	870
λ [W/m K]	3.1
ρ [kg/m^3]	2300

The borehole depth (H) is 100 meters and the radius (r_{bhe}) is 0.05 m, giving a ratio r_{bhe}/H of 0.0005. The borehole spacing B is 5 m, resulting in a B/H ratio equal to 0.05. A sketch of the geometry is shown in Figure 1. No distance under the borehole field has been considered in this case (further studies will include the effects of heat transfer under the borehole field).

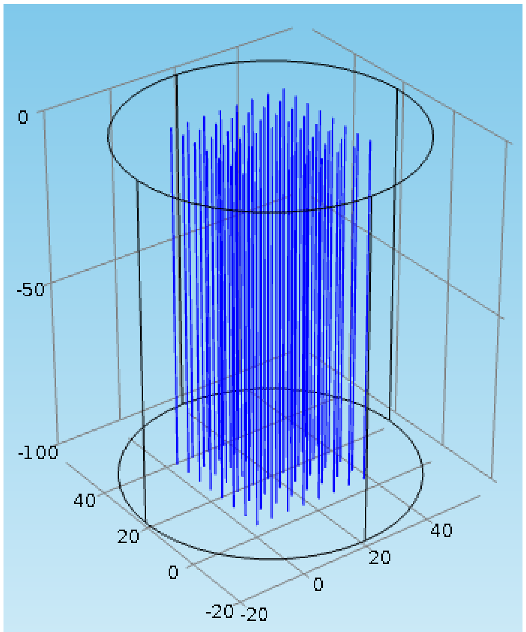

Figure 1. Geometry of the 8x8 borehole field

Simulations are carried out using different locations of the outer boundary in order to evaluate the sensitivity of the g-function calculation to the size of the model. The simulated outer radiuses are 40, 47.5, and 50 m.

The mesh is free triangular and the boundaries are defined at the top and the bottom of the sub-domain, later including a swept mesh of the radial elements that gives 20 elements along the depth of the sub-domain. The numbers of elements in the mesh is 97880, 99700, and 318160 for the 40, 47.5 and 50 m outer radius.

A heat flux at all borehole walls is imposed as a boundary condition, being set to 1 W/m, and an undisturbed ground temperature equal to 8°C is set far enough from the borehole field, at the outer radius of the simulated domain. The surface at the upper boundary of the geometry is considered at two different conditions: keeping it adiabatic, and later having a constant temperature of 8°C. The initial condition in the whole domain is also 8°C.

The average borehole wall temperature along the borehole depth (and for all boreholes) has been calculated at 30 days time steps for different amount of years, depending on the simulated case. The average temperature at the borehole wall is then written in terms of the dimensionless g-function, as expressed in Eq. (3), and plotted versus the natural logarithm of $9Fo_H$.

The model is solved until values of $\ln(9Fo_H)$ close to 2 are reached, where typically the asymptotic part of the g-function is reached. This Fourier value corresponds to about 200 years in our case. The simulations lasted between 30 to 110 minutes depending on the size of the geometry, using a 8 GB RAM Intel® Core ™ i5-2410M CPU 2.30 GHz.

All results are compared with a solution generated using the FLS method and the EED commercial software, using the same geometry.

4. Results and discussion

Figure 2 shows the g-function when the boundary condition of the surrounding undisturbed ground is set at a radius of 40 m from the center of the borehole field using the adiabatic boundary condition at the ground surface. The simulation is carried out for a time horizon of 20 and 200 years.

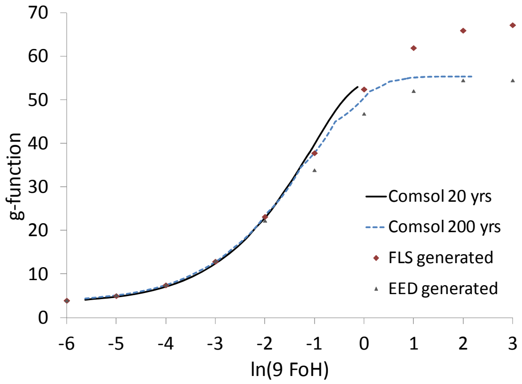

Figure 2. Comsol generated g-function, simulated for 20 and 200 years, using the outer boundary condition at a radius of 40 m, an adiabatic ground surface. Comparison with FLS and EED generated g-function

As seen in Figure 2, when the study is carried out for 20 yrs, the asymptotic part of the g-function is not achieved. In the 200 yrs case, the asymptotic g-function value is about 55 and it matches better the EED generated case. However, there is a better agreement with FLS up to $\ln(9Fo_H)\approx 0$.

The borehole field outer radius was then increased to 47.5 m, a limit set by the Comsol 4.2 itself (same results were later obtained using Comsol 4.3). Version 4.3 also allowed to further increase the dimension of the geometry. The asymptotic g-function value reached with these simulations resulted to be higher, as expected. The result is presented in Figure 3.

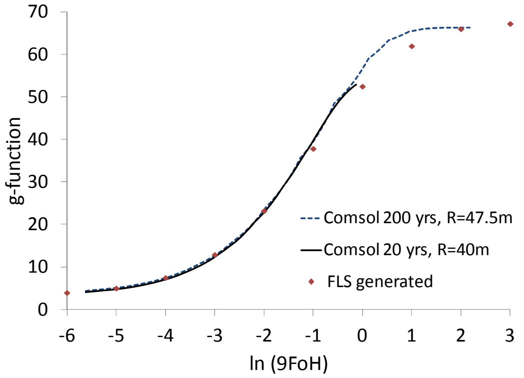

Figure 3. Comsol generated g-function, simulated for 20 yrs at a radius of 40 m and for 200 yrs at a radius of 47.5 m, using an adiabatic ground surface. Comparison with FLS generated g-function.

The difference between the Comsol results in the interval $-1<\ln(9Fo_H)<0$ observed in Figure 2 is not observed in Figure 3. The difference is attributed to the cooling effect of the 40 m outer boundary condition which keeps a constant and lower temperature during the whole simulation period (i.e. Figure 3 shows that all Comsol g-functions are now the same during the first 20 years, $\ln(9Fo_H)\approx 0$).

Even if some differences between the Comsol and FLS estimations can be observed in the range of $-1<\ln(9Fo_H)<2$, the general agreement in this case is good.

Actually, the location of the outer boundary in the Comsol model can be tuned in order to match different asymptote values, as illustrated in Figure 4 where a case where the outer boundary located at a 50 m boundary is compared to the previous results for a period of 200 years.

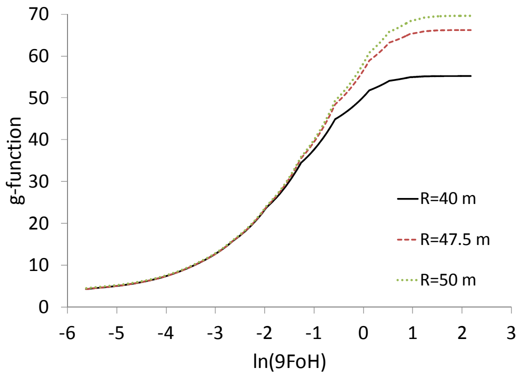

Figure 4. Comsol generated g-function, simulated for 200 years, using the outer boundary condition at a radius of 40, 47.5, and 50 m as well as an adiabatic ground surface

Both Comsol and FLS appear to overestimate the EED generated solution presented in Figure 2. It is worth mentioning at this point that the Comsol, FLS, and EED generated solutions presented until now in this paper actually use different boundary conditions.

Further numerical work will be done by the authors in the direction of assessing the influence of the boundary conditions as well as on the direction of increasing the physical domain dimensions (e.g. the outer radius, volume under the borehole field, etc). As a start point of this work, two simulations settings a constant temperature T_{surf}=8°C as a boundary condition at the ground surface have been performed. Figure 5 compares this with the previous results.

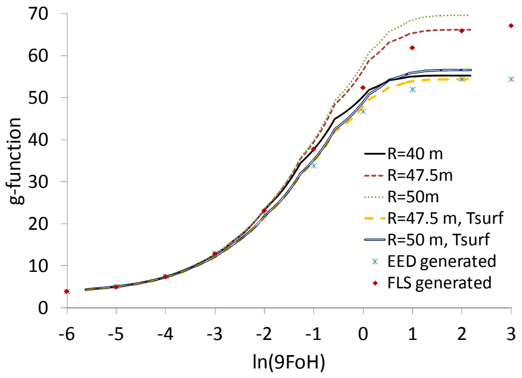

Figure 5. Comsol generated g-functions using adiabatic and constant temperature boundary condition at the ground surface, with outer boundary radius of 47.5 and 50 m. Comparison with FLS and EED generated g-function

As inferred from Figure 5, accounting for the ground surface temperature at the upper domain boundary (which is a better representation of the reality), radically changes the shape of the g-function. The effect of the outer ground boundary is still present but, in these cases, the solution gives a better match to the EED generated g-function. The FLS solution does, however, also account for this boundary.

6. Conclusions

A number of Comsol simulations have been performed for calculating the dimensionless temperature response factor (g-function) of a borehole rectangular arrangement composed of 64 heat exchangers.

The numerical results have been compared with those obtained by spatial superposition of the Finite Line Source (FLS) and to those generated using the EED software, generally showing good agreement, especially in the time range of -6<ln($9Fo_H$)<-1.

Further in time, 0<ln($9Fo_H$)<2, the Comsol model revealed to be influenced either by the domain dimensions, the simulation end time, and on whether the ground surface temperatures are taken into account.

Future investigation will be devoted to parametric analysis on simulation boundary conditions and domain geometry.

7. References

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