Numerical Study of Navier-Stokes Equations in Supersonic Flow over a Double Wedge Airfoil using Adaptive Grids

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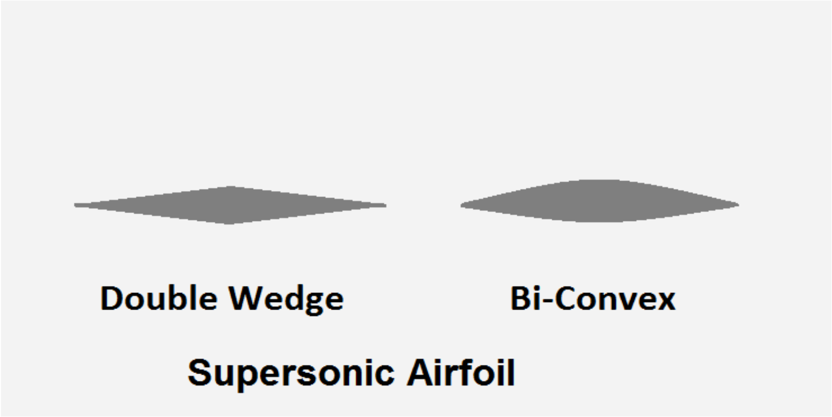
Supersonic Flight

Shock wave formed on supersonic flight (Courtesy:Ensign John Gay, US Navy)

Supersonic Airfoils



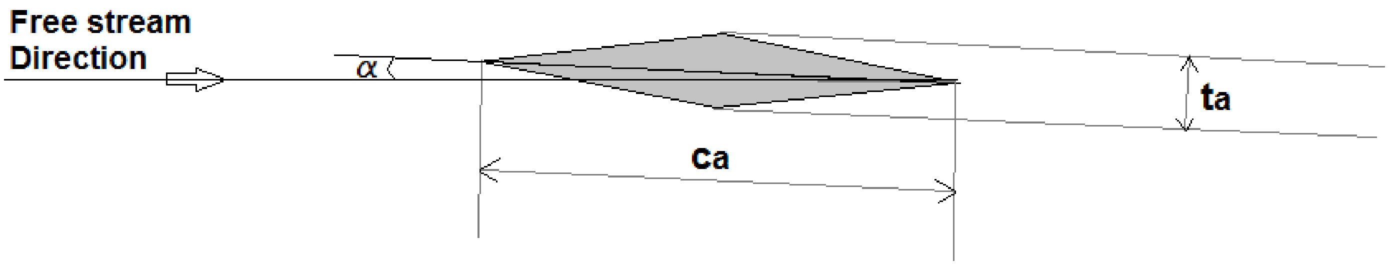
Supersonic Airfoil

- Thinner cross-section
- Sharper leading and trailing edge

Subsonic Airfoil

- Thicker cross-section
- Rounded leading and trailing edge

Symmetrical Double Wedge Airfoil



$$t_c = \frac{t_a}{c_a}: \{0.08, 0.1 \text{ and } 0.12\}$$

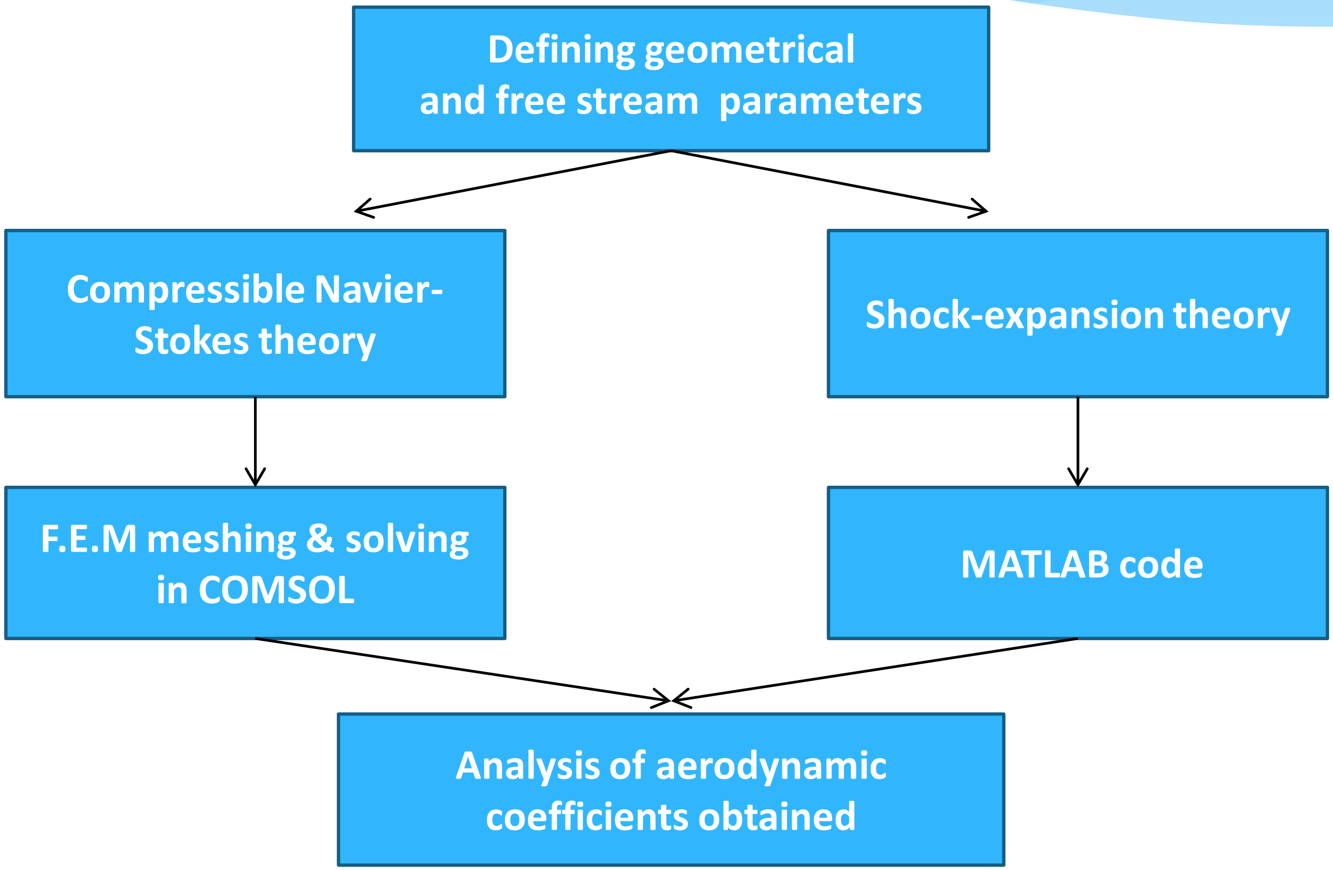
$$\alpha: \{0^0, 1^0, 2^0, 3^0, 4^0, 5^0, 8^0 \text{ and } 12^0\}$$

- c_a: Chord Length
- t_a: Thickness
- α : Angle of attack

Aerodynamic Coefficients

- Section aerodynamic coefficients of an airfoil is defined below:
  - Coefficient of pressure $C_p = \frac{p - p_\infty}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2}$
  - Coefficient of Lift $C_L = \frac{L}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot c}$
  - Coefficient of Drag $C_D = \frac{D}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot c}$

Evaluation of Aerodynamic coefficients



Compressible Navier-Stokes Theory

Non-conservative form:

Mass Conservation:

$$\frac{\partial \rho}{\partial t} + \rho \cdot \nabla \cdot (\boldsymbol{u}) + (\boldsymbol{u} \cdot \nabla)\rho = 0$$

Momentum Conservation:

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right) = \nabla \cdot \left[-p \cdot \boldsymbol{I} + \mu \cdot \left((\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T - \frac{2}{3} \cdot \nabla \cdot \boldsymbol{u} \cdot \boldsymbol{I}\right)\right]$$

Temperature equation:

$$\rho \cdot C_p\left(\frac{\partial T}{\partial t} + (\boldsymbol{u} \cdot \nabla)T\right) = \nabla \cdot (k \cdot \nabla T) + \frac{T}{\rho} \cdot \left(\frac{\partial \rho}{\partial T}\right)_p \left(\frac{\partial p}{\partial t} + (\boldsymbol{u} \cdot \nabla)p\right)$$
$$+ \nabla \boldsymbol{u} : \left(\mu \cdot \left((\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T - \frac{2}{3} \cdot \nabla \cdot \boldsymbol{u} \cdot \boldsymbol{I}\right)\right)$$

Ideal gas formulation:

$$p = \rho \cdot R \cdot T$$

Numerical Simulation

- <u>Boundary and Initial conditions</u>:

Free stream parameters	Domain inlet values
Mach numbers (M_∞)	2.5
Temperature (T_∞)	218 K
Pressure (P_∞)	0.2 atm

Domain outlet condition:

$$\nabla T \cdot \boldsymbol{n} = 0$$

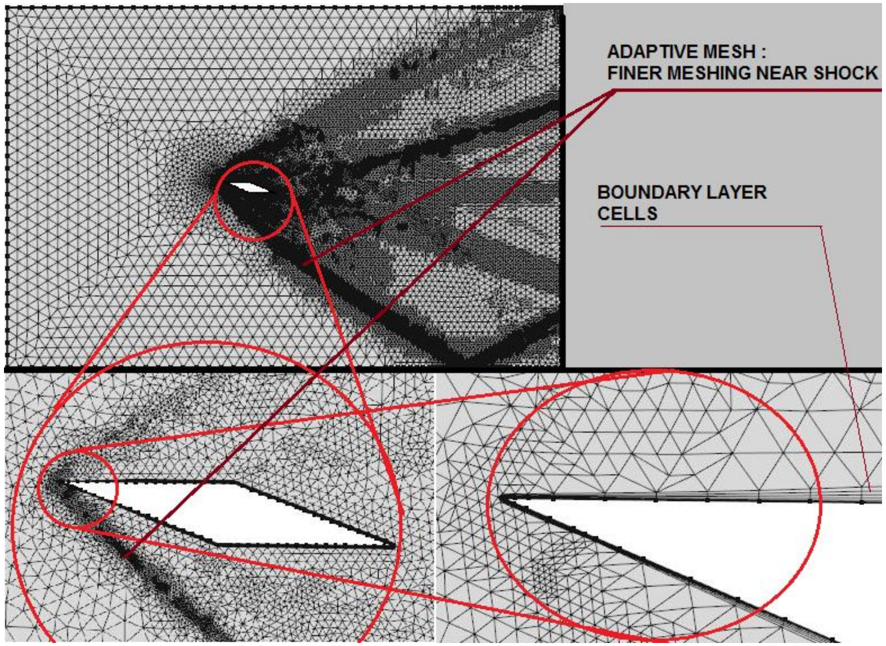
Initial domain parameters	Initial domain values for Euler computation
Pressure(p)	0.2 atm
Mach Number(M)	2.5
Temperature(T)	218 K

Grid generation

- Two level adaptive meshing feature on unstructured triangular mesh with first order element is implemented.
- Boundary-Layer cells are added to grid obtained from adaptive meshing.

Solver

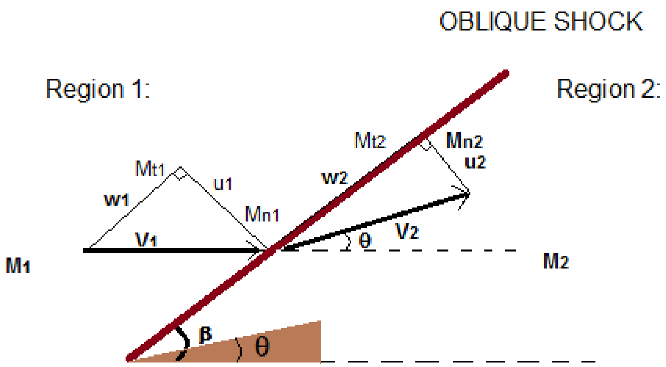
- Viscous computation is initialised with prior solution obtained from Euler equations
- All the primary variables while are fully coupled and are solved using pseudo time stepping with a stationary solver.
- The convergence was determined by setting the relative tolerance to 0.01.



Adaptive mesh with boundary layer cells for Adaptive grid for $t_c = 0.1\, and\, \alpha = 12^0$ case.

Shock-Expansion Theory

Oblique Shock



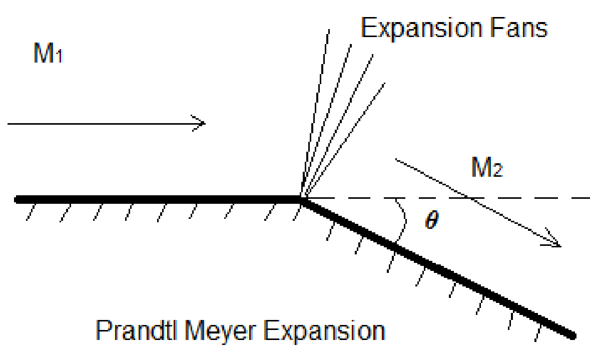
$\beta - \theta - M$ Relation

$$\tan(\beta) = \frac{\left(M_1^2 - 1 + 2\cdot\lambda\cdot\cos\left[\left(4\cdot\pi\cdot\delta + \frac{\cos^{-1}(\chi)}{3}\right)\right]\right)}{3\cdot\left(1 + \frac{(\gamma-1)}{2}\cdot M_1^2\right)\cdot\tan(\theta)}$$

$$\lambda = \left[\left(M_1^2 - 1\right)^2 - 3\cdot\left(1 + \frac{(\gamma-1)}{2}\cdot M_1^2\right)\cdot\left(1 + \frac{(\gamma+1)}{2}\cdot M_1^2\right)\cdot\tan^2(\theta)\right]^{\frac{1}{2}}$$

$$\chi = \frac{\left(\left(M_1^2 - 1\right)^3 - 9\cdot\left(1 + \frac{(\gamma-1)}{2}\cdot M_1^2\right)\cdot\left(1 + \frac{(\gamma-1)}{2}\cdot M_1^2 + \frac{(\gamma+1)}{4}\cdot M_1^4\right)\cdot\tan^2(\theta)\right)}{\lambda^3}$$

Prandtl-Meyer Expansion



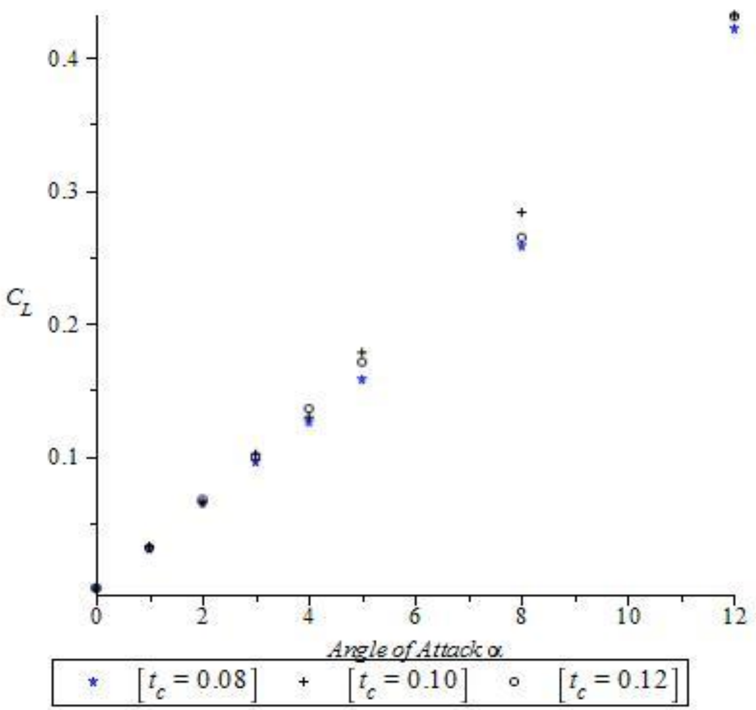
Isentropic expansion equation

$$\theta = f(M_2) - f(M_1)$$

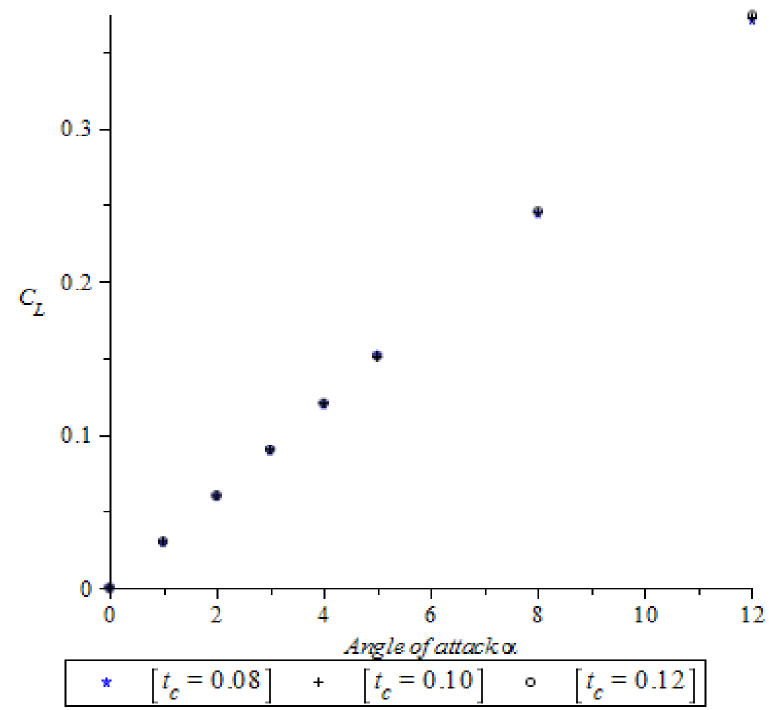
$$f(M) = \sqrt{\frac{(\gamma+1)}{(\gamma-1)}}\cdot\tan^{-1}\left(\sqrt{\frac{(\gamma-1)}{(\gamma+1)}\cdot\left(M^2 - 1\right)}\right) - \tan^{-1}\left(\sqrt{M^2 - 1}\right)$$

Results

Coefficient of Lift

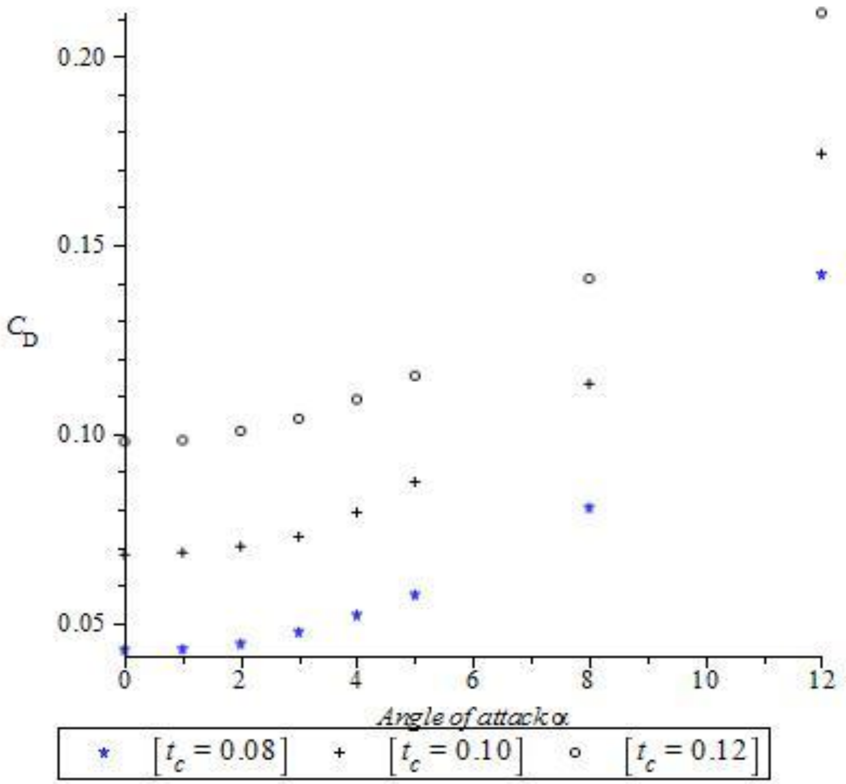


$C_L v/s\ \alpha$ (F.E.M Simulation)

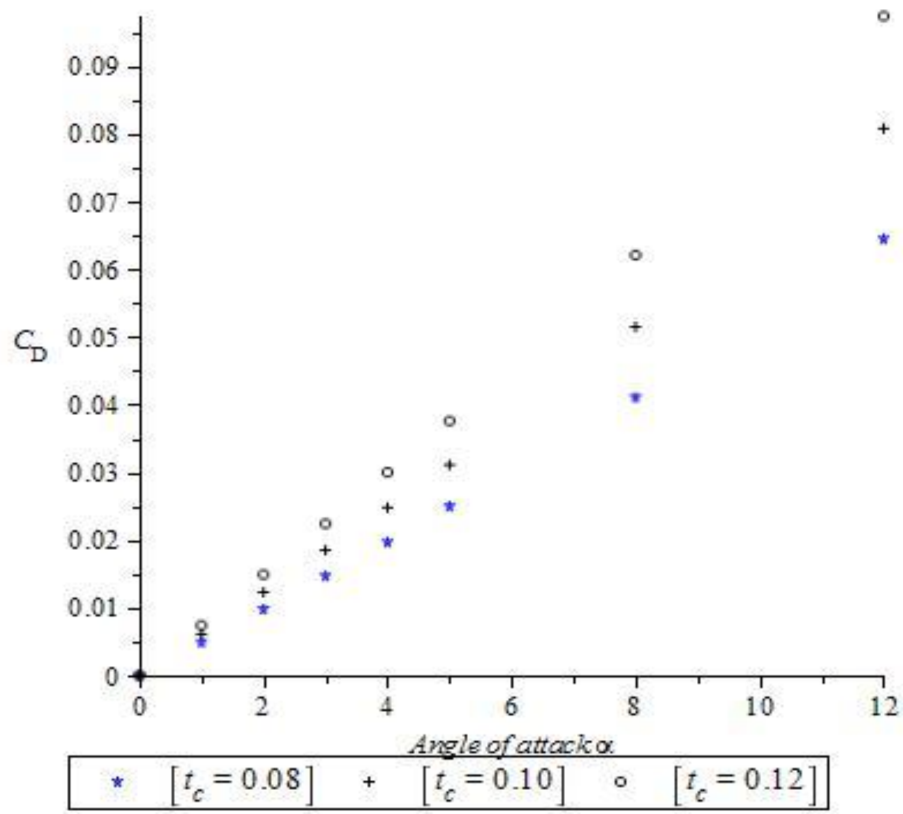


$C_L v/s\ \alpha$ (SE-theory)

Coefficient of Drag



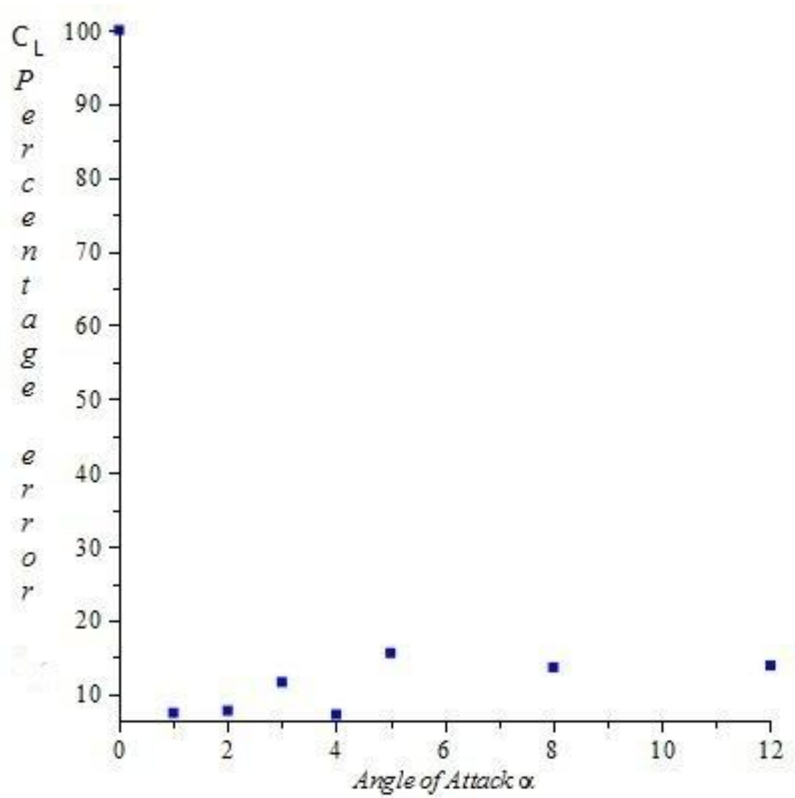
$C_D v/s\ \alpha$ (F.E.M Simulation)



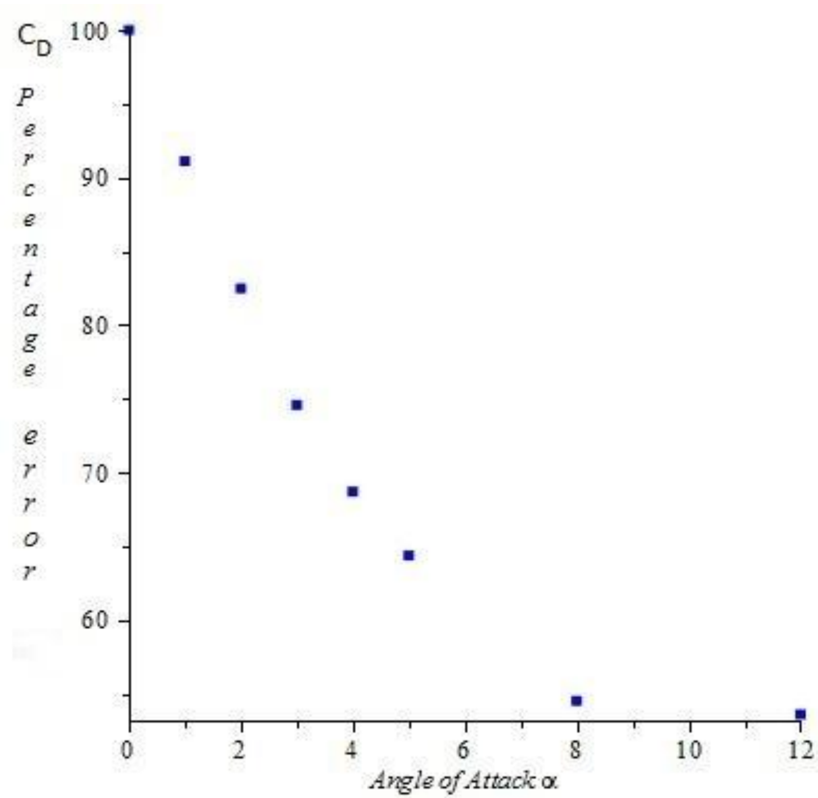
$C_D v/s\ \alpha$ (SE-theory)

Percentage Error estimation for ($t_c = 0.1$)

$$\frac{FEM - SE}{FEM} X100$$

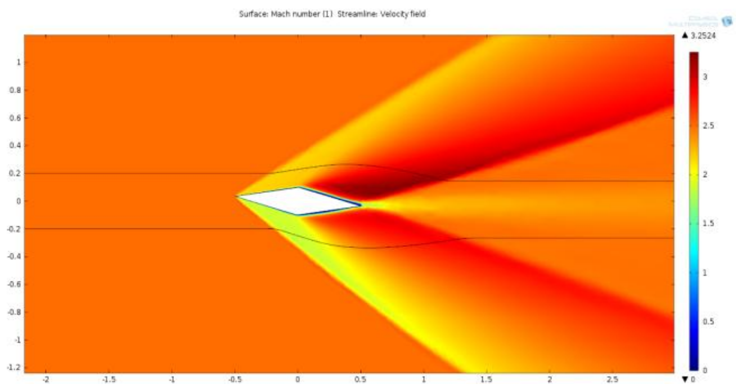


Percentage error of C_L v/s α

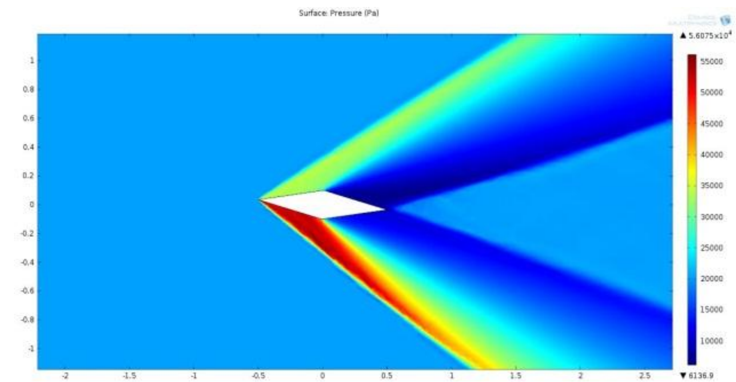


Percentage error of $C_D v/s$ α

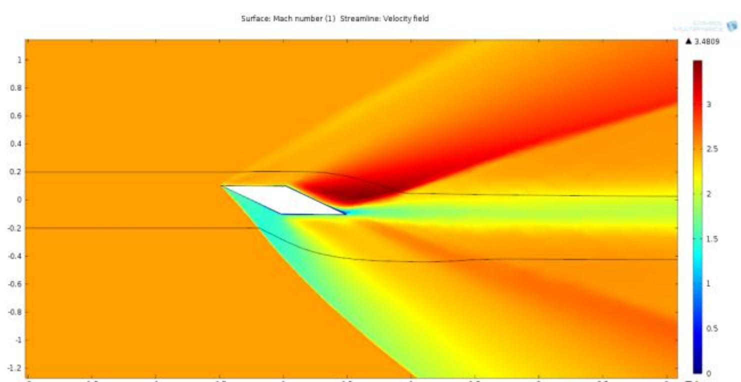
- <u>Results for specific case</u> ($t_c = 0.1$):



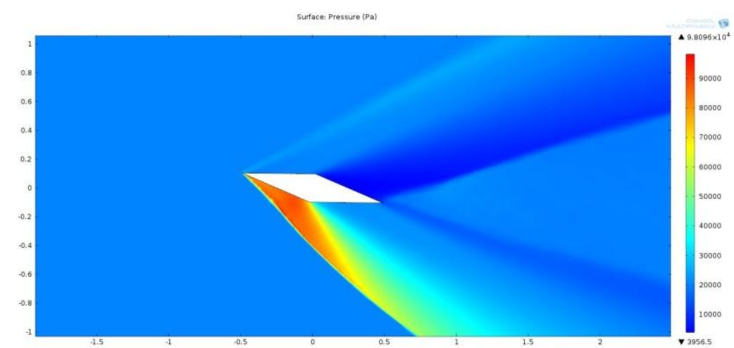
Mach number plot at $\alpha = 4$



Pressure plot at $\alpha = 4$

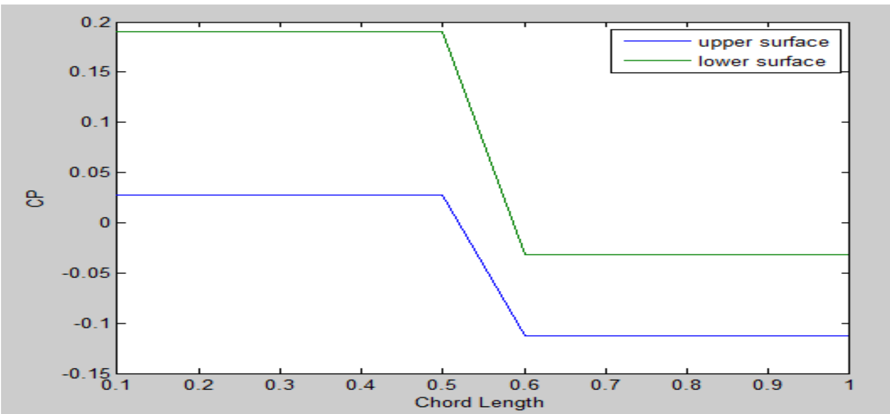


Mach number plot at $\alpha = 12$

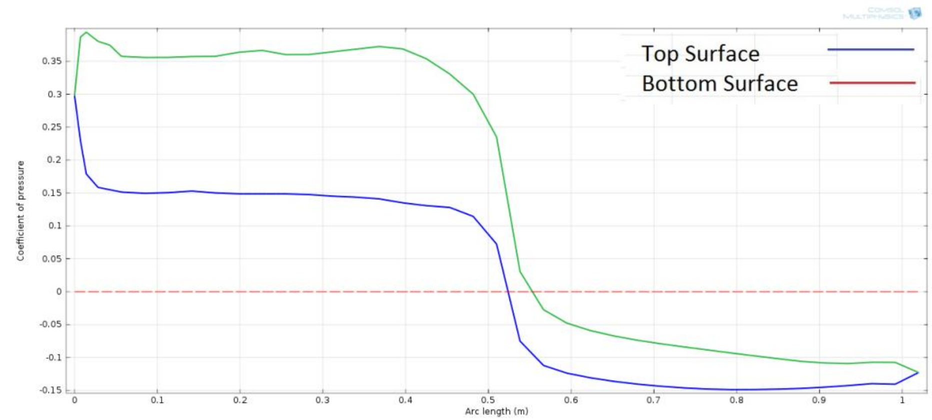


Pressure plot at $\alpha = 12$

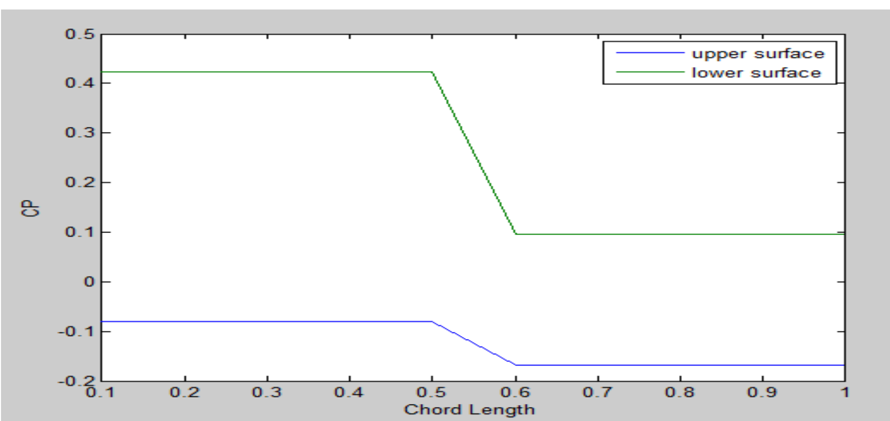
Coefficient of pressure C_p $(t_c = 0.1)$



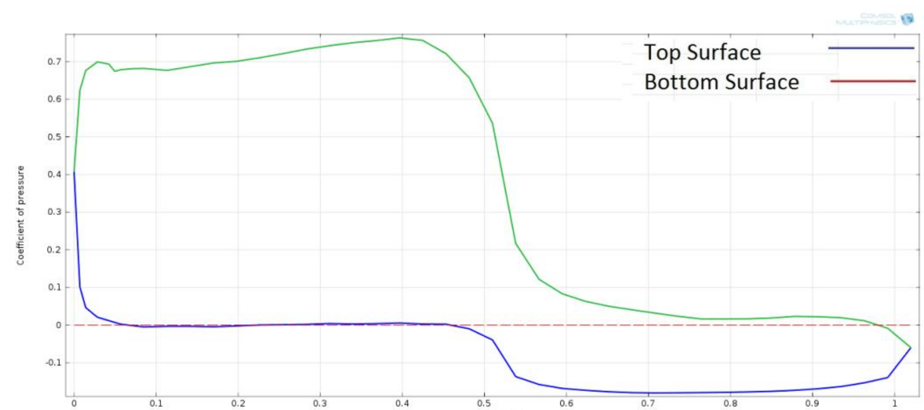
C_P (SE-theory, $\alpha = 4$)



C_P (F.E.M, $\alpha = 4$)



C_P (SE-theory, $\alpha = 12$)



C_P (F.E.M, $\alpha = 12$)

CONCLUSION

- The solutions obtained from numerical simulation performed with FEM tool is in good agreement with shock-expansion theory.

- The difference in the values of coefficients obtained from SE-theory and compressible NS numerical simulation indicates the expected viscous and wake effects.

- The values of coefficients obtained from F.E.M simulation are only applicable for infinite span wing having airfoil section congruent to aerofoil designed in this current work.

THANK YOU

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QUERIES ?