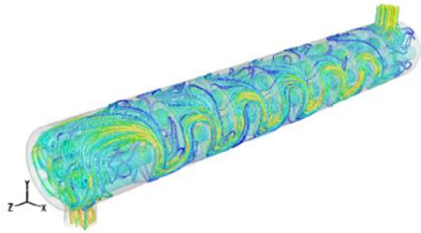


# Diffusion and Reaction in Fe-Based Catalyst for Fischer-Tropsch Synthesis Using Micro Kinetic Rate Expressions

## 3-D CFD Model for Shell & Tube Exchanger with 7 Tubes



Ender Ozden and Ilker Tari (2010)

**Arvind Nanduri & Patrick L. Mills\***

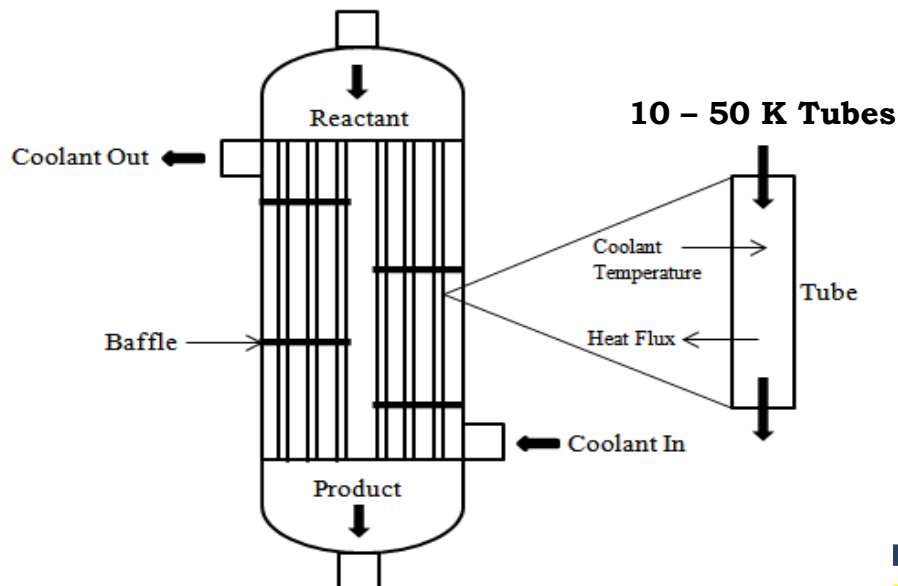
**Department of Chemical & Natural Gas Engineering**

**Texas A&M University-Kingsville**

**Kingsville, TX 78363-8202 USA**

**\*Patrick.Mills@tamuk.edu**

## Multitubular Reactor Design for Low Temperature Fischer-Tropsch



**COMSOL  
CONFERENCE  
2014 BOSTON**

**Session: Transport Phenomena  
October 9, 2014**

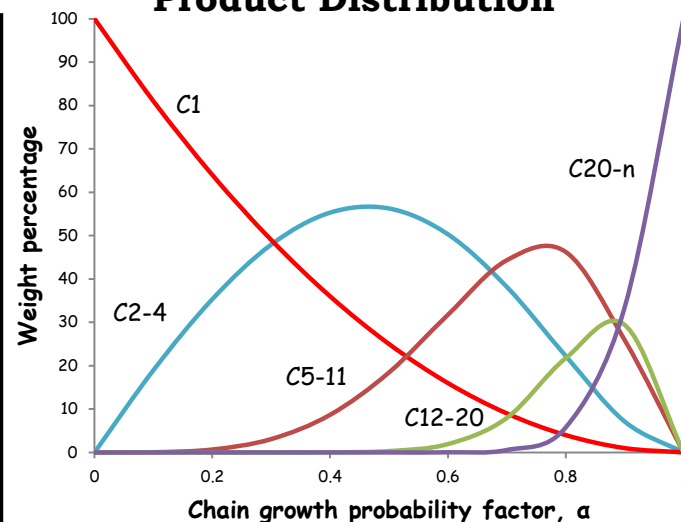


**TEXAS A&M  
UNIVERSITY  
KINGSVILLE**

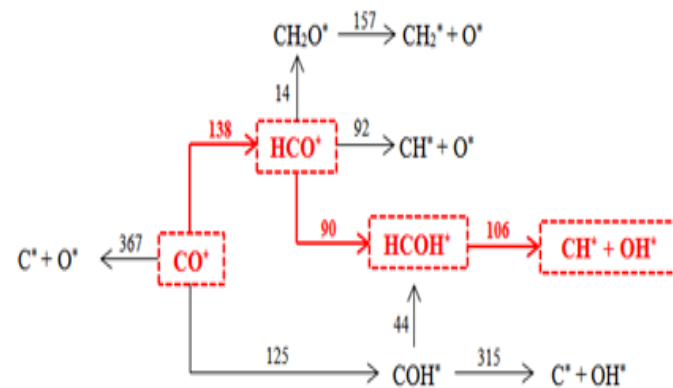
# Presentation Outline

- Introduction
- Objectives
- F-T Chemistry, Kinetics & Thermo
- Multiphysics Model Equations
- Key Results
  - Catalyst Performance
  - Concentration Profiles
  - Computational Difficulties
- Conclusions

## Anderson-Schulz-Flory (ASF) Product Distribution



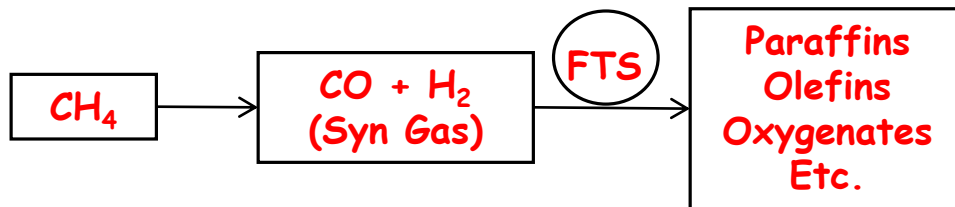
## CO Dissociation Pathway



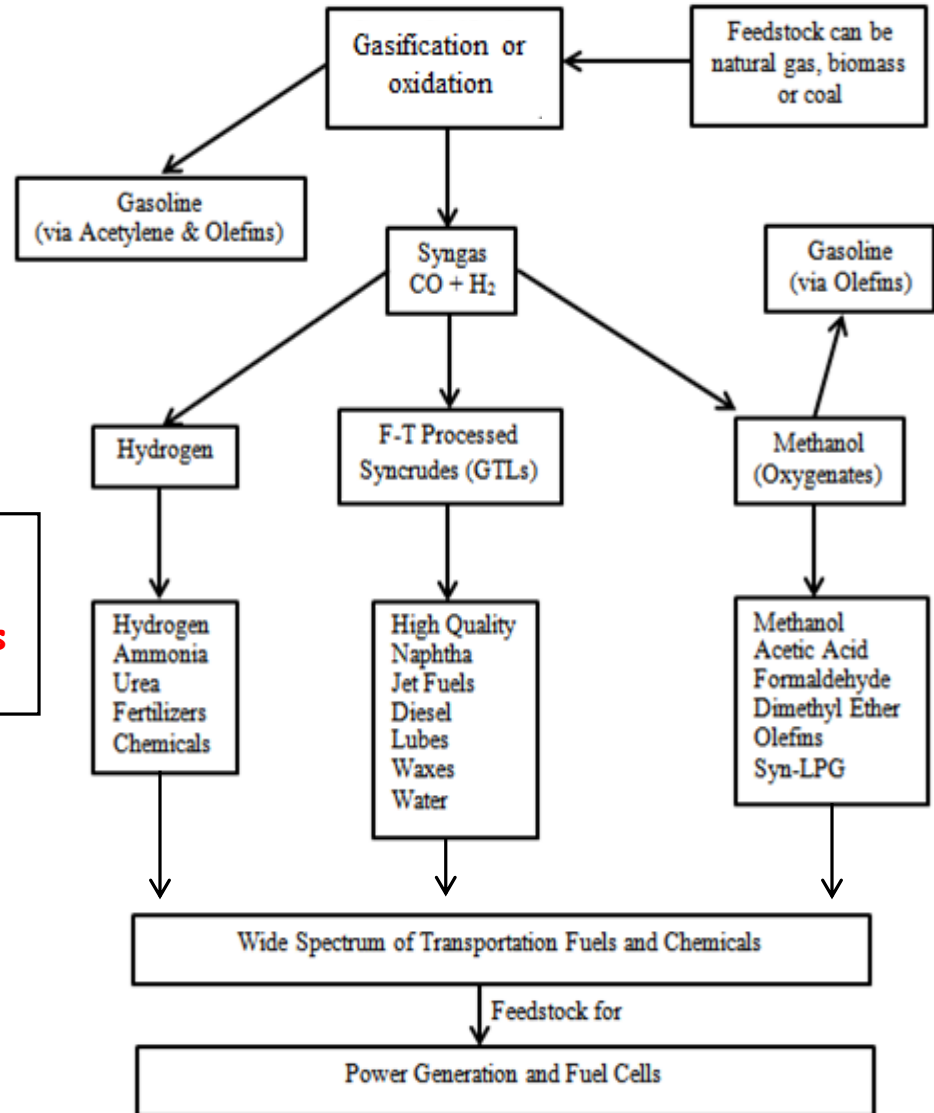
M. Ojeda *et al.* (2008)

# Introduction

- Fischer-Tropsch synthesis (FTS) is a highly exothermic polymerization reaction of syngas ( $\text{CO} + \text{H}_2$ ) in the presence of Fe/Co/Ru-based catalysts to produce a wide range of paraffins, olefins and oxygenates, often known as *syncrude*

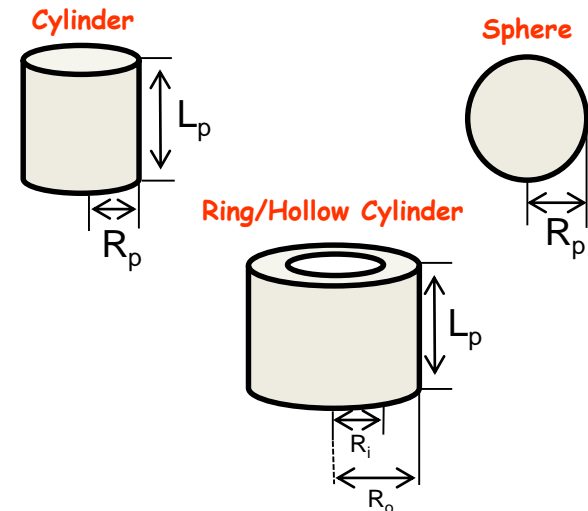
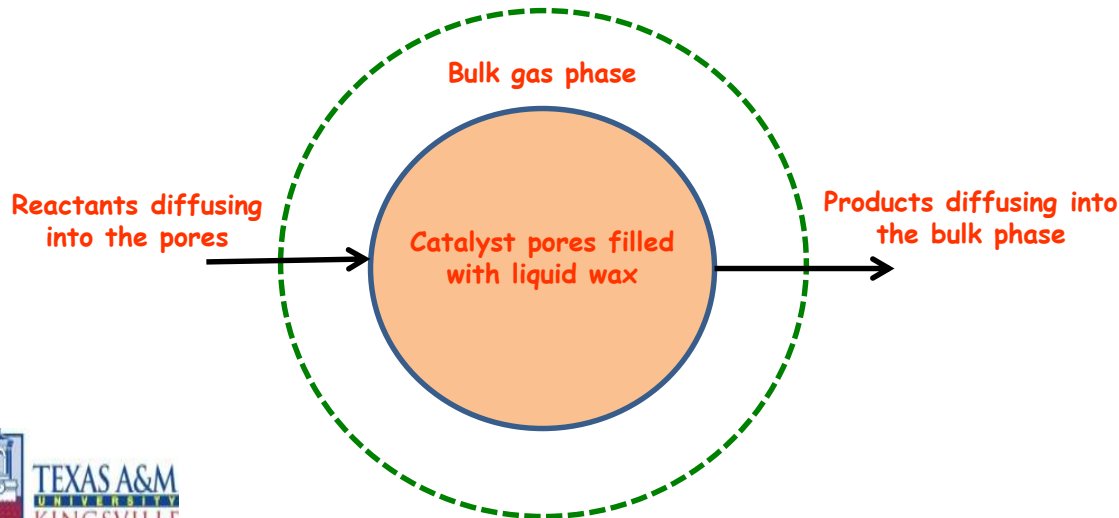


- Standard large-scale gas conversion
- Isolated "Stranded gas" conversion



# Objectives

- Model the Fischer-Tropsch (FT) reaction network
  - Implement micro-kinetic rate expressions
  - Assess the effect of process parameters on the FT product distribution
    - i. Catalyst particle shape
    - ii. Operating conditions (T, P)
- Incorporate Soave-Redlich-Kwong (SRK) equation of state (EOS) into the particle-scale transport-kinetics model to more accurately describe the vapor-liquid-equilibrium (VLE) behavior of the FT product distribution within the porous catalyst particle.



# Key F-T Catalytic Reactions

Main Reactions		
1	Methane	$CO + 3H_2 \rightarrow CH_4 + H_2O$
2	Paraffins	$(2n+2) H_2 + n CO \rightarrow C_nH_{2n+2} + n H_2O$
3	Olefins	$2n H_2 + n CO \rightarrow C_nH_{2n} + n H_2O$
4	WGS (only on Fe catalyst)	$CO + H_2O \leftrightarrow CO_2 + H_2$
Side Reactions		
5	Alcohols	$2n H_2 + n CO \rightarrow C_nH_{2n+1}O + n H_2O$
6	Boudouard Reaction	$2CO \rightarrow C + CO_2$
Catalyst Modifications		
7	Catalyst Oxidation/Reduction	(a) $M_xO_y + y H_2 \rightarrow y H_2O + x M$ (b) $M_xO_y + y CO \rightarrow y CO_2 + x M$
8	Bulk Carbide Formation	$y C + x M \rightarrow M_xC_y$

## Conventional Names of F-T Products

Name	Composition
Fuel Gas	$C_1-C_2$
LPG	$C_3-C_4$
Gasoline	$C_5-C_{12}$
Naphtha	$C_8-C_{12}$
Kerosene	$C_{11}-C_{13}$
Diesel/Gasoil	$C_{13}-C_{17}$
F-T Wax	$C_{20+}$

# Fischer-Tropsch Micro-kinetic Rates

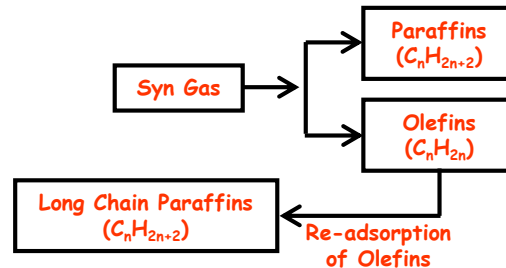
## Fe-Based Olefin Readsorption Microkinetic Model

$$R_{CH_4} = \frac{k_{5M} P_{H_2} \alpha_1}{1 + \left( 1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4} \right) \sum_{i=1}^N (\prod_{j=1}^i \alpha_j)}$$

$$R_{C_n H_{2n+2}} = \frac{k_5 P_{H_2} \prod_{j=1}^n \alpha_j}{1 + \left( 1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4} \right) \sum_{i=1}^N (\prod_{j=1}^i \alpha_j)}$$

$$R_{C_n H_{2n}} = \frac{k_6 (1 - \beta_n) \prod_{j=1}^n \alpha_j}{1 + \left( 1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4} \right) \sum_{i=1}^N (\prod_{j=1}^i \alpha_j)}$$

$$R_{CO_2} = \frac{k_v \left( \frac{P_{CO} P_{H_2O}}{P_{H_2}^{0.5}} - \frac{P_{CO_2} P_{H_2}^{0.5}}{K_p} \right)}{1 + \frac{K_v P_{CO} P_{H_2O}}{P_{H_2}^{0.5}}}$$



$$\beta_n = \frac{\frac{k_{-6}}{k_6} P_{C_n H_{2n}}}{\left[ \alpha_A^{i-2} \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2}} + \frac{k_{-6}}{k_1 P_{CO} + k_5 P_{H_2} + k_6} \sum_{i=2}^n \left( \alpha_A^{i-2} P_{C_{(n-i+2)} H_{2(n-i+2)}} \right) \right]}$$

$$\alpha_n = \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2} + k_6 (1 - \beta_n)}$$

$$\alpha_A = \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2} + k_6}$$

$$K_p = \exp \left[ \frac{5078.0045}{T} - 5.8972089 + 13.958689 \right. \\ \left. * 10^{-4} T - 27.592844 * 10^{-8} T^2 \right]$$

$n = 2$  to  $20$

Wang et al., *Fuels* (2003)

# Thermodynamics of F-T Reaction Mixtures

## Soave-Redlich-Kwong (SRK) EOS

$$P_i = \frac{RT}{(V_i - b_i)} - \frac{\alpha_i a_i}{V_i(V_i + b_i)}$$

$$Z_i^3 - Z_i^2 + Z_i(A_i - B_i - B_i^2) - A_i B_i$$

$$A_i = \frac{\alpha_i P_i}{R^2 T^2}$$

$$\alpha_i = 0.42747 \frac{R^2 T_{ic}^2}{P_{ic}}$$

$$B_i = \frac{b_i P_i}{RT}$$

$$b_i = 0.08664 \frac{RT_{ic}}{P_{ic}}$$

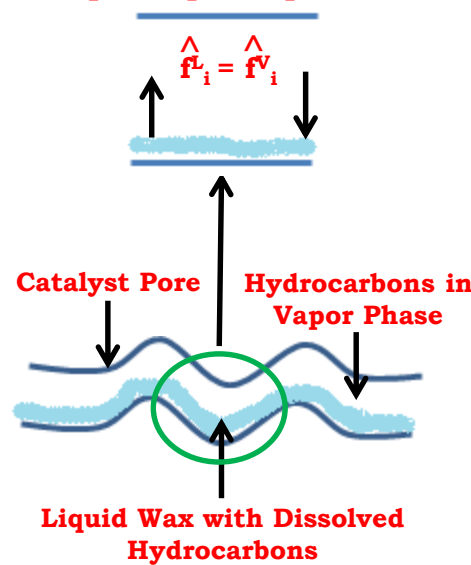
$$\alpha_i = \left(1 + m_i(1 - \sqrt{T_{ir}})\right)^2$$

$$m_i = 0.48508 + 1.55171\omega_i - 0.1561\omega_i^2$$

$$\ln \phi_i^P = \frac{b_i}{b_m} (Z_i - 1) - \ln(Z_i - B_i) + \frac{A_i}{B_i} \left( \frac{b_i}{b_m} - \frac{2}{\alpha_i a_i} \sum_j y_j (\alpha_i a_i)_{ij} \right) \ln \left( 1 + \frac{B_i}{Z_i} \right)$$

$$a_m = \sum_i \sum_j y_i y_j (a_i a_j)^{1/2} (1 - k_{ij})$$

### Vapor-Liquid Equilibrium



Liquid Wax with Dissolved Hydrocarbons

$$b_m = \sum_i y_i b_i$$

## Flash Calculations

### Rachford-Rice Objective Function

$$F(\alpha_g) = \sum_i \frac{z_i (K_i - 1)}{(1 + \alpha_g (K_i - 1))} = 0$$

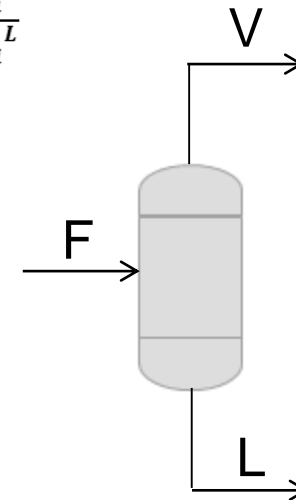
$i = 1$  to 43 with 43 distinct roots

Only the positive roots less than 1 are used for VLE calculations

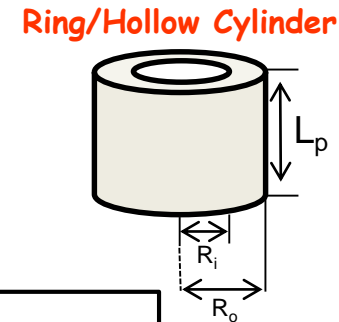
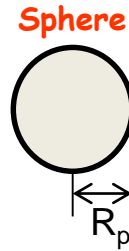
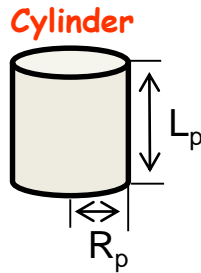
### Wilson's Correlation

$$K_i^{\text{guess value}} = \frac{P_{ic}}{P} \exp \left( 5.37(1 + \omega_i) \left( 1 - \frac{T_{ic}}{T} \right) \right)$$

$$K_i = \frac{\phi_i^V}{\phi_i^L}$$



# Catalyst Properties & Process Conditions



$$\text{Volume}_{\text{sphere}} = \text{Volume}_{\text{cylinder}} = \text{Volume}_{\text{ring}}$$

$$(4/3) R_{\text{sphere}}^3 = L_{\text{cylinder}} R_{\text{cylinder}}^2 = L_{\text{ring}} (R_o^2 - R_i^2)$$

## Dimensions of Cylinder and Ring for $R_{\text{sphere}} = 1.5 \text{ mm}$

Cylinder	$L = 3 \text{ mm} \ \& \ R = 1 \text{ mm}$
Ring	$L = 2 \text{ mm}, \ R_o = 1.5 \text{ mm} \ \& \ R_i = 0.3 \text{ mm}$

## Catalyst Properties

Density of pellet, $\rho_p$	$1.95 \times 10^6 \text{ (gm/m}^3\text{)}$
Porosity of pellet, $\epsilon$	0.51
Tortuosity, $\tau$	2.6

## Dimensions of Cylinder and Ring for $R_{\text{sphere}} = 1 \text{ mm}$

Cylinder	$L = 3 \text{ mm} \ \& \ R = 0.7 \text{ mm}$
Ring	$L = 2 \text{ mm}, \ R_o = 1.5 \text{ mm} \ \& \ R_i = 1 \text{ mm}$

## Operating Conditions

Temperature, $^{\circ}\text{K}$	493, 523 & 533
Pressure, bar	20, 25 & 30
$\text{H}_2/\text{CO}$	2



# Governing Multiphysics Model Equations

43 species and 43 reactions

**General Species Balance:**  $\nabla \cdot (-D_{ei} \nabla C_i) = \rho_p \sum_j \alpha_{ij} R_{ij}$

**Species Balance for Spherical Catalyst Particle:**  $\frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( D_{ei} \xi^2 \frac{\partial C_i}{\partial \xi} \right) = -\rho_p R_p^2 \sum_j \alpha_{ij} R_{ij}$   
 where,  $\xi = r / R_p$

**Species Balance for Cylindrical Catalyst Particle:**  $\frac{1}{\xi} \frac{\partial}{\partial \xi} \left( D_{ei} \xi \frac{\partial C_i}{\partial \xi} \right) = -\rho_p R_p^2 \sum_j \alpha_{ij} R_{ij}$   
 where,  $\xi = r / R_p$

**Species Balance for Ring Catalyst Particle:**  $\frac{1}{(\xi \delta + R_i)} \frac{\partial}{\partial \xi} \left( (\xi \delta + R_i) D_{ei} \frac{\partial C_i}{\partial \xi} \right) = -\rho_p \delta^2 \sum_j \alpha_{ij} R_{ij}$   
 where,  $\xi = (r - R_i) / (R_o - R_i)$  &  $\delta = R_o - R_i$

**Effective Diffusivity:**  $D_{ei} = \frac{\epsilon D_{iB}}{\tau}$  ( $\epsilon$  = porosity and  $\tau$  = tortuosity)

$$D_{CO_2,B} = 5.584 * 10^{-7} e^{\left(\frac{-1786.29}{T}\right)}$$

$$D_{H_2,B} = 1.085 * 10^{-6} e^{\left(\frac{-1624.63}{T}\right)}$$

$$D_{CO_2,B} = 3.449 * 10^{-7} e^{\left(\frac{-1613.65}{T}\right)}$$

**Molecular Diffusivities of Hydrocarbons in Wax**

$$D_{i,B} = D_{CO_2,B} \left( \frac{V_{CO}}{V_i} \right)^{0.6}$$

$V$  = molar volume

# Model Assumptions & Boundary Conditions



## Boundary Conditions

<b>Spherical Particle</b>	At $\xi = -1$ and $\xi = 1$ , $C_i = C_{i,bulk}$ ( $CO_{2,bulk} = \epsilon ps$ for convergence)
<b>Cylindrical Particle</b>	At $\xi = -1$ and $\xi = 1$ , $C_i = C_{i,bulk}$ ( $CO_{2,bulk} = \epsilon ps$ for convergence)
<b>Ring Particle</b>	At $\xi = 0$ and $\xi = 1$ , $C_i = C_{i,bulk}$ ( $CO_{2,bulk} = \epsilon ps$ for convergence)

### Species Flux

- Independent of composition  $C_i$
- Dependent on local temperature  $T$
- Future work: Use multicomponent flux transport models

### COMSOL Modules

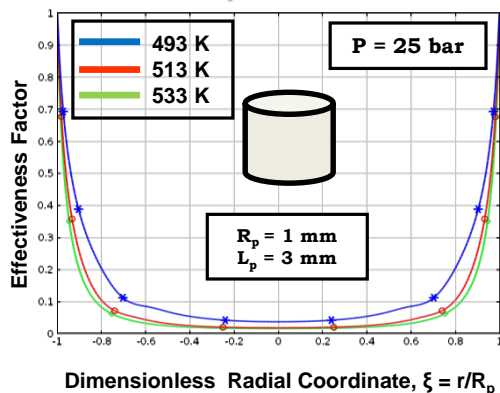
- Transport of Diluted Species
- Coefficient Form PDE Solver

### Key Assumptions

- i. Concentration is a function of only the radial coordinate, *i.e.*,  $C_i = C_i(r)$
- ii. Steady-state
- iii. All catalyst particle shapes have the same material properties ( $\epsilon$ ,  $\tau$ ,  $\rho$ ,  $k_{eff}$ )
- iv. Isothermal conditions (since  $\Delta T$  is small)
- v. Bulk gas phase contains only  $H_2$  and  $CO$  (Reactor entrance conditions)

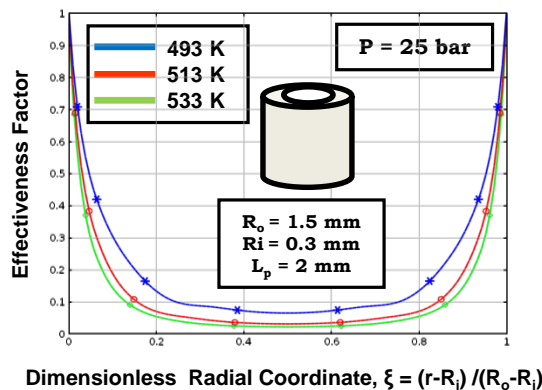
# Various Catalyst Shapes: $\eta$ & $C_i$ Profiles

## Cylinder



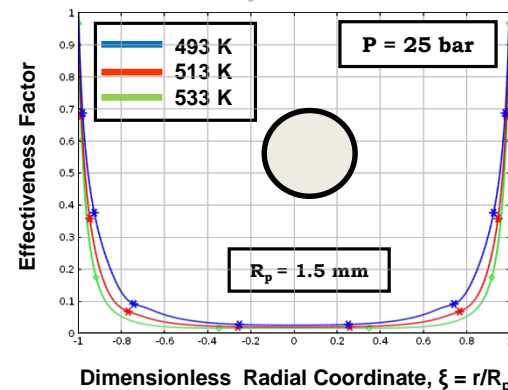
Dimensionless Radial Coordinate,  $\xi = r/R_p$

## Ring/Hollow Cylinder

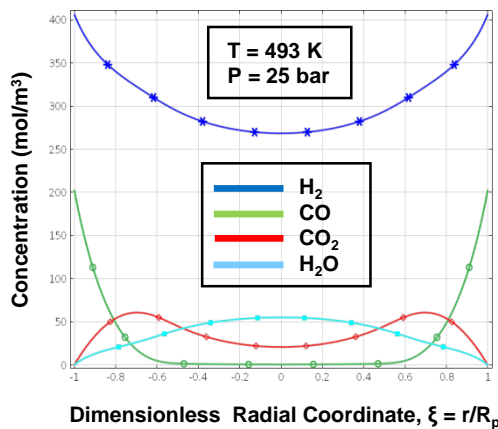


Dimensionless Radial Coordinate,  $\xi = (r-R_i)/(R_o-R_i)$

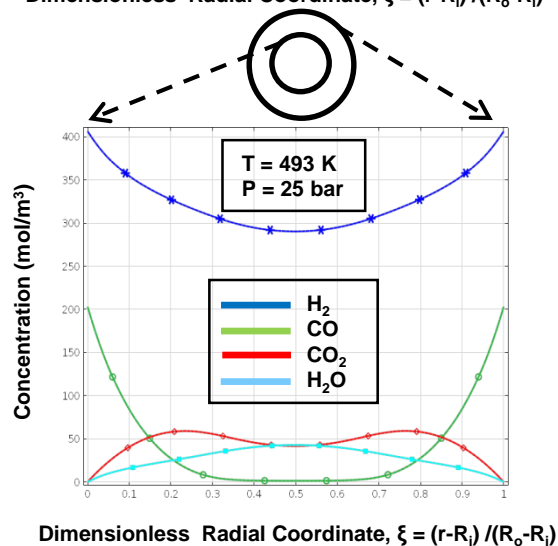
## Sphere



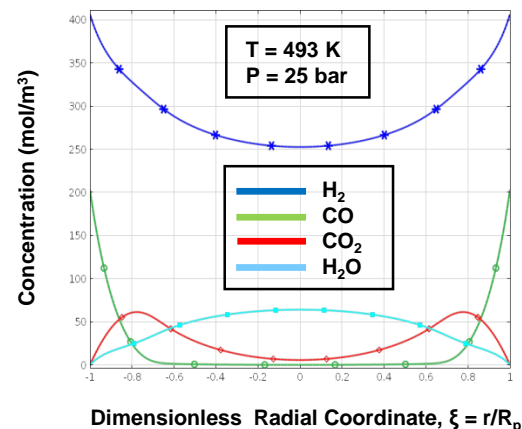
Dimensionless Radial Coordinate,  $\xi = r/R_p$



Dimensionless Radial Coordinate,  $\xi = r/R_p$

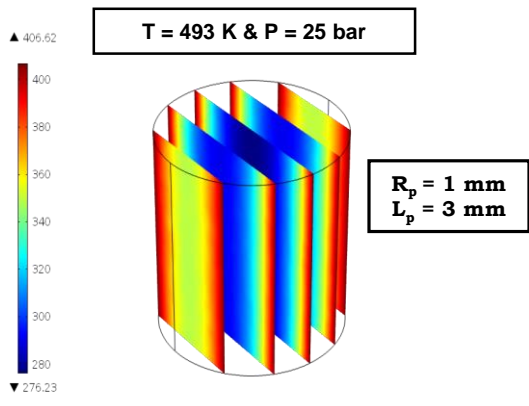


Dimensionless Radial Coordinate,  $\xi = (r-R_i)/(R_o-R_i)$

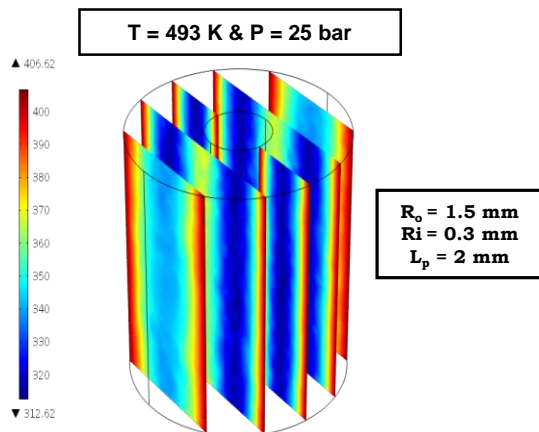


Dimensionless Radial Coordinate,  $\xi = r/R_p$

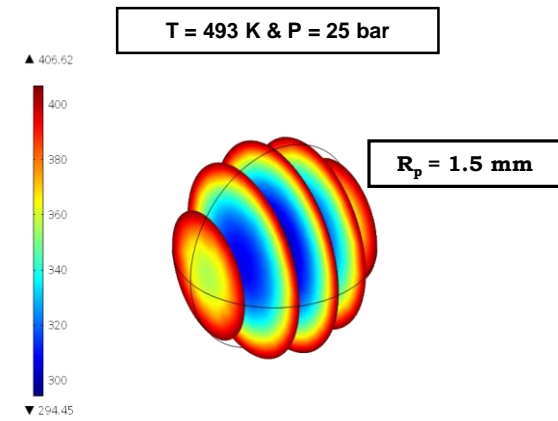




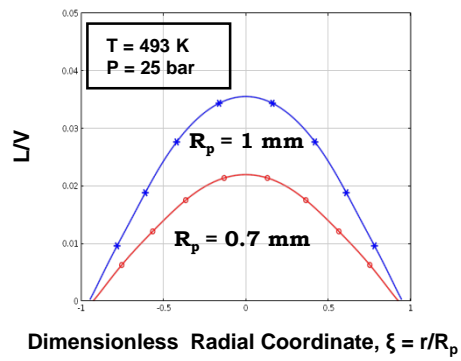
H<sub>2</sub> Concentration Profile



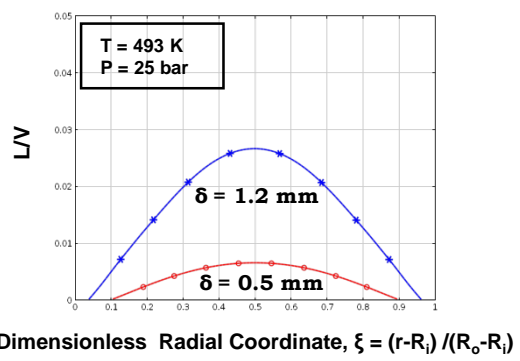
H<sub>2</sub> Concentration Profile



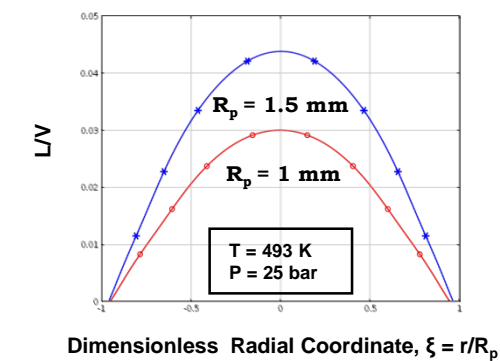
H<sub>2</sub> Concentration Profile



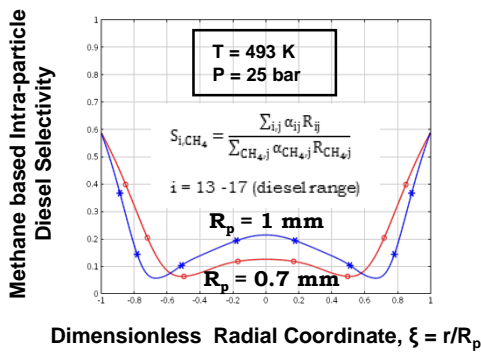
Dimensionless Radial Coordinate,  $\xi = r/R_p$



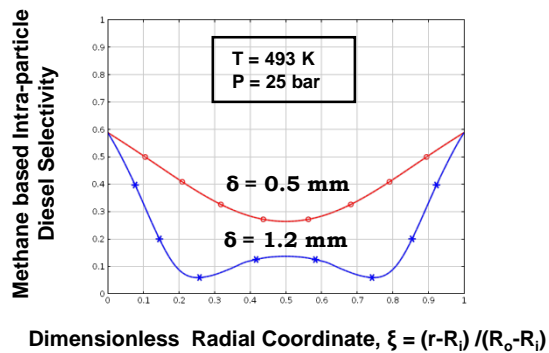
Dimensionless Radial Coordinate,  $\xi = (r-R_i)/(R_o-R_i)$



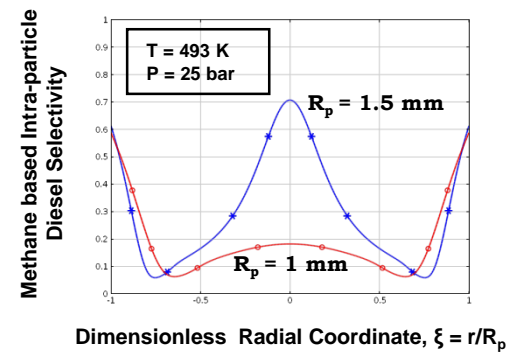
Dimensionless Radial Coordinate,  $\xi = r/R_p$



Dimensionless Radial Coordinate,  $\xi = r/R_p$

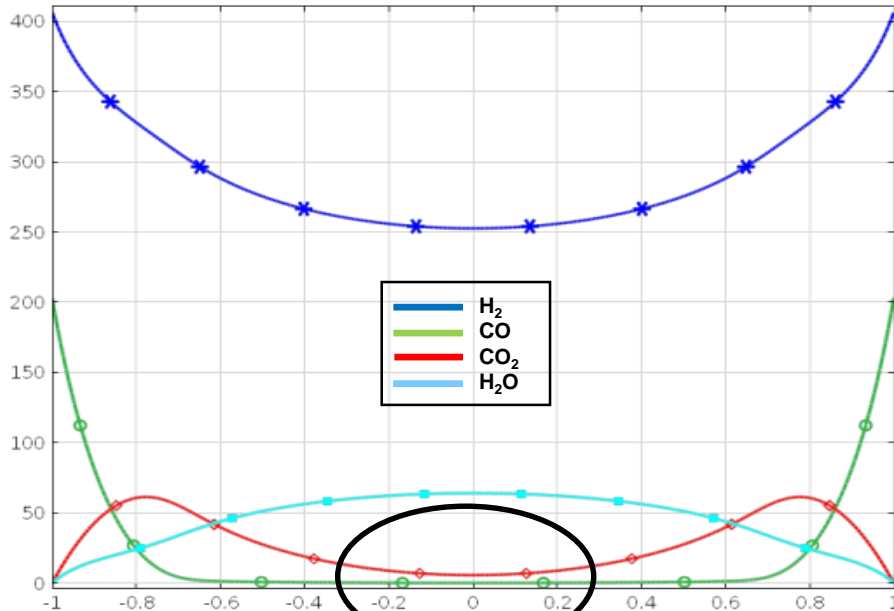


Dimensionless Radial Coordinate,  $\xi = (r-R_i)/(R_o-R_i)$



Dimensionless Radial Coordinate,  $\xi = r/R_p$

# Computational Issues



**Region with numerical instabilities**

Once the convergence issue was solved the mesh was refined to get smooth curves.

- To avoid convergence issues, the radius of the particle was set to a very small number and the subsequent solution was stored to be used as initial conditions for higher radius.
- Numerical instabilities were encountered in the region where CO and CO<sub>2</sub> concentrations approached zero leading to convergence issues and unrealistic values.
- The convergence issues were solved by not letting CO and CO<sub>2</sub> concentrations approach zero by using  $CO = \text{if}(CO \leq 0, \text{eps}, CO)$  and  $CO_2 = \text{if}(CO_2 \leq 0, \text{eps}, CO)$ .

# Conclusions

- A 1-D catalyst pellet model can be used to analyze particle-level performance. Catalyst performance on a reactor-scale can be studied by coupling the pellet model to the tube & shell-side models for the MTFBR.
- The CO conversion, effectiveness factor, intra-particle liquid to vapor (L/V) fraction, catalyst strength and the diesel selectivity results suggest that the cylindrical and spherical catalyst particle shapes are preferred over hollow rings. The presence of more liquid in the spherical particle creates an advantage for the cylindrical catalyst shape due to diffusional limitations in the wax.
- Micro kinetic rate equations, when coupled with intraparticle transport effects and vapor-liquid equilibrium phenomena, captures the transport-kinetic interactions and phase behavior for gas-phase FT catalysts.
- Convergence can be a major issue in fast reaction-diffusion systems. This can sometimes be easily resolved by using simple built-in operators, such as 'if ()' and 'eps', to avoid negative and other unrealistic values of dependent variables at the boundaries or interior and then refining the mesh in accordance with computational time.

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Thank You





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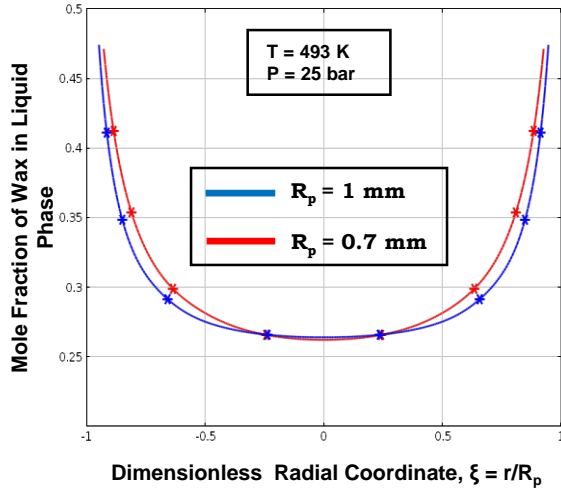
# References (cont'd)

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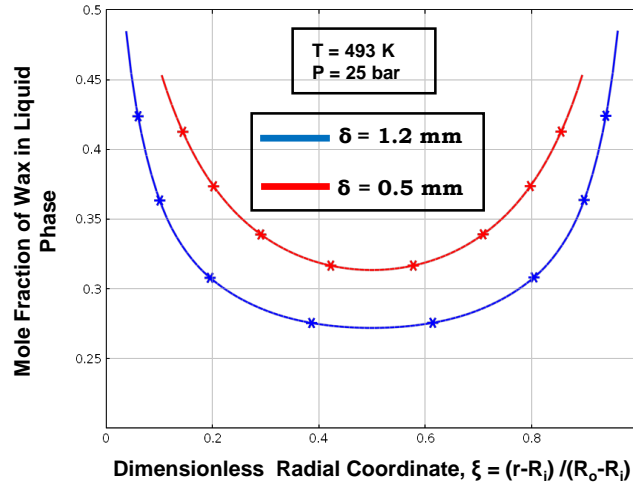
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- [17] J. Eilers, S. A. Posthuma and S. T. Sie, "The SHELL Middle Distillate Synthesis Process (SMDS),"

# Mole Fraction of Wax & Diesel in Liquid Phase

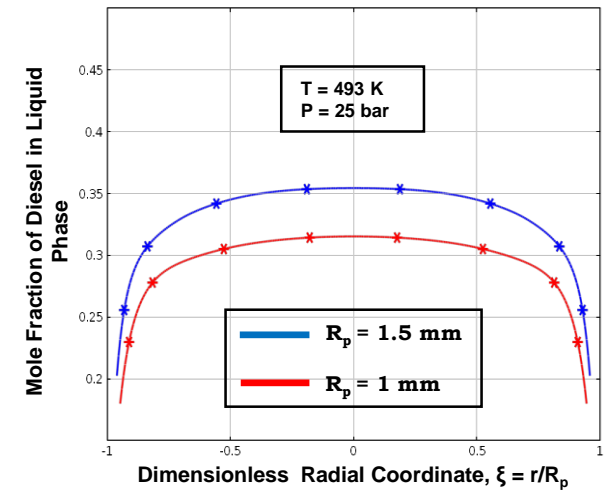
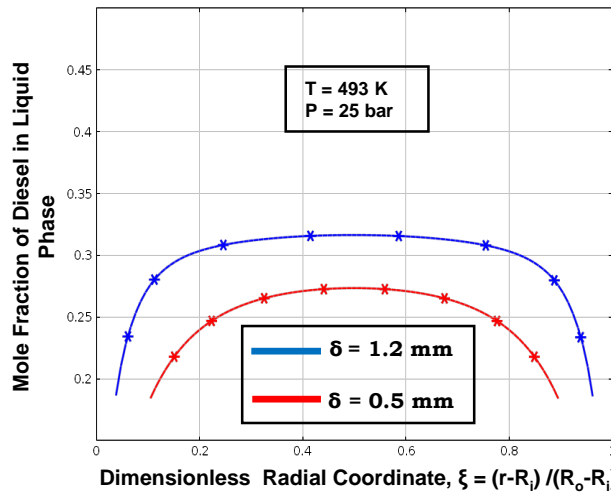
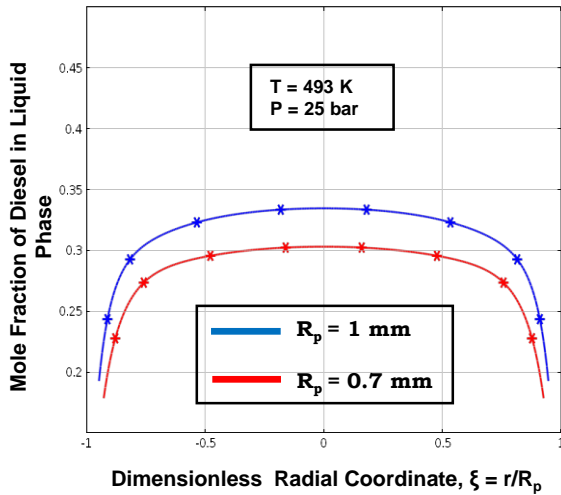
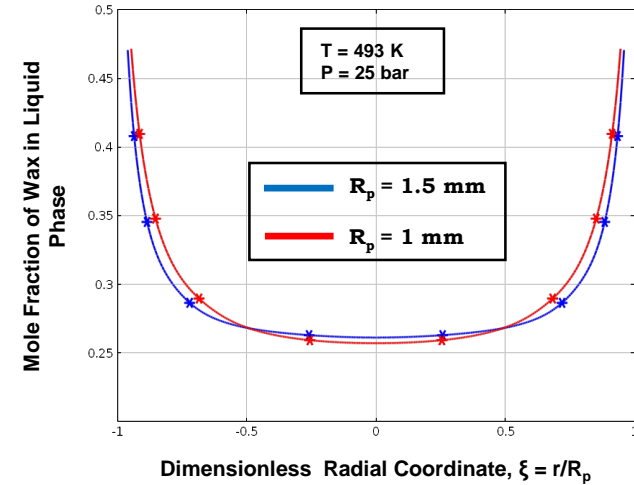
## Cylinder



## Ring/Hollow Cylinder

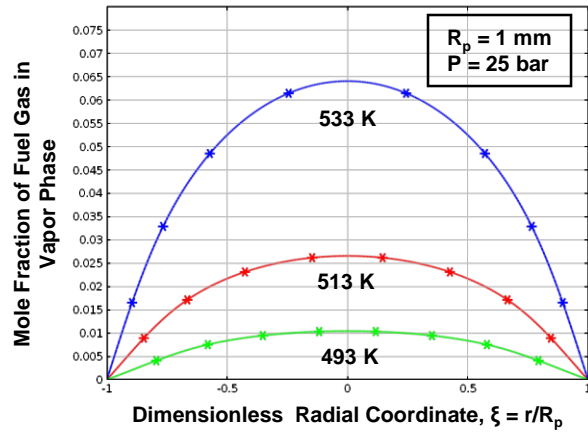


## Sphere

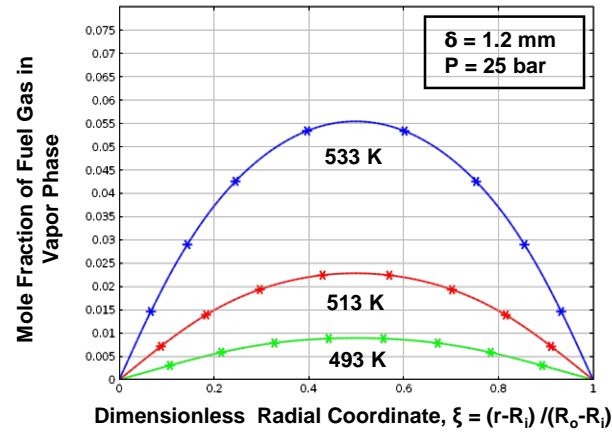


# Mole Fraction of Fuel Gas in Vapor Phase

## Cylinder



## Ring/Hollow Cylinder



## Sphere

