Teaching Simulation Methods with COMSOL Multiphysics®

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Abstract: This paper describes two courses on simulation methods for graduate students: "Simulation Methods" and "Simulation and Optimization in Virtual Engineering" The courses were planned to teach young engineers how to work with simulation software as well as to understand the necessary mathematical background. As simulation software COMSOL is used. The main philosophy was to combine theory and praxis in a way that motivates the students. In addition "soft skills" should be improved. This was achieved by project work as final examination. As underlying didactical principle the ideas of Bloom's revised taxonomy were followed. The paper basically focusses on educational aspects, e.g. how to structure the course, plan the exercises, organize the project work and include practical COMSOL examples.

Keywords: COMSOL Multiphysics**,** Optimization module, LiveLink for MATLAB, Bloom Taxonomy, education

1. Introduction

Simulation of components and processes becomes more and more important in industry, even in medium-sized enterprises [1, 2]. Therefore it is necessary to train young engineers how to apply simulation software in a sensible way, so that they can take advantage of the full potential of it. For that reason it is important to provide special courses on simulation methods in engineering education [1, 3, 4].

In addition, it also becomes essential to learn "soft skills" e.g. teamwork, interdisciplinary work, project management, presentation techniques and literature search [5]. This is not only demanded by public institutions [6, 7], but also by industry executives [8].

Based on these circumstances two consecutive elective courses on simulation methods in the master degree program "Energy Systems" were established at the FH Aachen. The focus was on COMSOL Multiphysics, because it is widely utilized in industry and very intuitively to handle [4, 9]. To achieve an excellent education for the students, different didactical methods were developed. Although the courses are placed in the field of energy technology, it is certainly possible to transfer all described methods and ideas to courses in other engineering fields.

The main philosophy behind the courses is to combine theory and practice, although it is clear that students do not like theory. They prefer more practical exercises and want to start immediately with the computer, because they want to "feel like engineers" [5]. To overcome this conflict, theory and practical exercises were mixed, following the ideas of the revised Bloom's Taxonomy [11] (See Sec. 2).

Instead of the typical written or oral exams, project work in groups was installed, because it is perfectly suitable to train "soft skills" and the learning achievement is much higher [12, 13].

The outline of the paper is as follows: In the next section (Sec. 2), the general setup of the courses is introduced. In the main section of the paper (Sec. 3) the applied teaching methods are described in detail, motivated by the revised Bloom's taxonomy. Further the realization and grading of the project work is carried out (Sec. 3.3).

Sec. 4 contains example projects in COMSOL from the courses. Finally the paper closes with the lecturers' and the students observations during the course and the project work (Sec. 5).

2. General Setup of the Course

2.1 Master Program "Energy Systems"

The courses were held at the Faculty of Energy Technology at the FH Aachen in the master degree program "Energy Systems".

It lasts four semesters (180 ECTS, 1st-3rd semester: lectures, 4th semester: thesis). The focus of the program is on energy technologies from a mechanical and electrical engineering point of view. Currently there are about 150 students in this program. It is an international program with students from all around the world (65%) and from Germany (35%).

2.2 Framework of the courses

Both courses are 5 ECTS points elective courses (15 weeks long). They are designed for students in the second ("Simulation Methods") and the third ("Simulation and Optimization in Virtual Engineering") semester. Then they already have completed the necessary compulsory courses and achieved the necessary knowledge in mathematics, physics and engineering.

"Simulation Methods" was held in the winter semester 2012/13 where nine students participated and winter semester 2013/14 with 21 students. "Simulation and Optimization in Virtual Engineering" was held in summer semester 2013 and 2014 with 16 students.

In the first 12 weeks, the courses consist of four times 45 minutes. These three hours split roughly into 1.5h lecture and 1.5h computer exercises, which are not strictly separated. Finally the last three weeks were reserved for project work.

Provided course material were the presented slides and detailed lecture notes in electronic form. The notes contain the theoretical background, exercises with solutions as well as descripttions for the computer exercises. The lecture was held by a professor, who was assisted by one engineer in the computer exercises (two engineers in the first course 2012/13).

2.3 Topics

An overview of the course topics is shown in Fig. 1. It is differed between physical and more mathematical topics. The focus in "Simulation Methods" was basically on heat transfer problems.

Since this paper deals with teaching methods, just a few comments to selected topics:

- **Numerical Solvers:** The Finite Difference Method is discussed, although COMSOL is based on Finite Element Methods. There are two reasons: First there is another elective course on FEM in the master program. Further it suffices, that the students get the basic ideas behind numerical solution of PDEs. Thus it is adequate to teach the simpler FDM.
- **Convection:** Convection is a complex process, which involves both, heat transport and fluid dynamics. Thus this topic was divided into three parts: First just conduction was considered, with predefined fluid flow, which is e.g. the case for pipe flows. Then the focus was on fluid dynamics without thermodynamical effects. Laminar as well as turbulent flow regimes were discussed. Then finally heat transport and fluid dynamics were coupled. This includes forced and free, natural convection.

3. Applied Teaching Methods

The organization of the lectures followed the revised Bloom's Taxonomy [11], which is one of the basic educational principles for the Bachelor and Master reform due to the Bolonia Process in Europe. The situation for "Simulation Methods" is displayed in Fig. 2. The levels rise during the semester. At the beginning the descriptions for the exercises are very detailed, whereas more and more facts are omitted in the later examples.

	Mathematical / Simulation Topics			Physical Topics		
	Basics on PDEs	**Geometry Handling**	**Numerical Solvers**	**Conduction**	**Radiation**	**Convection**
Simulation Methods	- Vector Analysis - Mathematical Formulation	- Import of CAD Structures - Geometry Builder in COMSOL	- Finite Difference Method - Numerical Linear Algebra - Newton's Method	- Mathematical Description - Boundary Conditions - Homogenization	-Stefan-Boltzmann Law	- Forced Convection - Free Convection - Fluid Dynamics
	Optimization Theory	**Multicriteria Optimization**	**Numercal Solution Methods**	**Heat Transfer**	**Structural Mechanics**	**Multiphysics**
Simulation and Optimization in Virtual Engineering	- Unconstrained Problems - Constrained Problems (KKT) - Convex Problems	- Optimizing two objective Functions - Weighted sum approach	- Decent, Steepest Decent Method -Penalty and Barrier Function Methods - Numerical Derivatives - Opt. with MATLAB	- Conduction - Convection - Radiation	- Mechanics of Materials - Eigenfrequencies	- Heat conduction and mechanical deform.

Figure 1. Overview on the course topics

This guaranteed that the students learn to work autonomous with the software, which is a main goal of the lecture.

The first three lectures are of 1st level, containing a short repetition and introduction. Later the lectures changed to 2nd and 3rd level, where the topics were more complicated and challenging. Finally the course closed with project work, which can be regarded as highest level in the lecture.

The following subsections contain detailed descriptions exemplarily for "Simulation Methods".

3.1 1st Level Lectures

The 1st level lectures were the introductory lectures (1-3 week). The goal was to provide the students with enough background knowledge for later topics.

First common facts on PDEs were repeated, which automatically directed to differential operators that were utilized to write down the corresponding PDEs (remember). Further these repetitions were combined with the second category: understand. Not only the definition of the PDEs and the differential operators were repeated, also their meaning was explained. For example: the gradient is perpendicular to the contour lines and the divergence is closely related to sources and sinks. Further these facts were related to physical phenomena and their mathematical description.

As new topic in this category, the mathematical formulation of physical processes via boundary value problems was discussed. The students already knew the physical background, but typically this is taught without higher mathe-

matics. The three possible types of boundary conditions for the heat equation were discussed: Dirichlet, Neumann and Robin boundary conditions. This included the mathematical notation via formulas, as well as the physical interpretation and practical realization. In addition these facts were related to the provided boundary conditions in COMSOL.

The theoretical parts of lectures were accompanied by practical parts (apply). The lectures started with simple heat conduction problems (e.g. a thermal bridge), which were related to the theoretical parts. In these introductory examples, a detailed instruction was provided, how to set up the simulations in COMSOL. As nice assistant in planning the lectures and especially the simulation examples, COMSOL provides a model library, which contains several practical examples for all covered physical phenomena. This includes detailed descriptions, instructions and already implemented simulations.

Finally two exercises were solved: In the first exercise a simple heat conduction problem was provided in mathematical form. Then the students were first asked to translate it into a physical problem. After that they had to solve the problem in COMSOL, without instructions. The problem in the second exercise was inverse to the first one: a physical explanation of a heat conduction problem was given and the students had to find the mathematical description. Again this repeated all topics by applying the acquired knowledge.

Structure of the Cognitive Process	1st Level (1.-3. week)	2nd Level (6. & 7. week)	3rd Level (11. & 12. week)	Project (13.-15. week)
Remember	Repetition PDEs; Differential op.;			Math., phys. and Comsol basics;
Understand	Differential op.; Heat equation; Boundary cond.;	Direct and iterative solvers;	Math. principle of natural conv.;	Practical problem;
Apply	Interpr. of math. formulation.; Application in Comsol.;	Example in Matlab; Transfer theory to Comsol;	Simulation series in Comsol;	Implementation in Comsol;
Analyze		Convergence criteria;	Criteria for convection and conduction;	Project schedule; Analyze results;
Evaluate			Diff. types of flow regimes;	Reviewing and judging the results;
Create				Putting results together; Creating report and presentation;

Figure 2. Oganization of the lectures, following the revised Bloom's Taxonomy.

3.2 2nd Level Lectures

As example for a later, 2nd level lecture, now the lectures on numerical linear algebra are considered (6.+7. week). As preparation, FDM for simple boundary value problems was already discussed.

The lecture started with the category: understand. Here first the different numerical methods for solving linear systems of equations were classified: direct and iterative.

Then the difference between both types was investigated. As result the students understood, that iterative methods were more appropriated.

In the next step, the students exemplary investigated the iterative Jacobi and Gauss-Seidel method and the SOR method, which are available in COMSOL. This part was connected to a practical exercise (apply): Two simple linear systems of equations were considered in Matlab and solved by the Jacobi method. As result only one method converged. This directly leaded to a mathematical analysis of convergence criteria (analyze). After deriving typical convergence critera, the students compared them with their observations. Further the new knowledge was directly related to the available COMSOL solvers and the possible manipulations.

3.3 3rd Level Lectures

Later in the course more and more 3rd level problems were considered. As example, now the teaching of natural convection is discussed (last two weeks).

The lectures started with the physical and mathematical principle of natural convection. First a general situation was considered, then several thought experiments were conducted (simple rectangulars in 2D). After that the lecture changed to the application category by considering once again the thought experiments, but now with COMSOL. The simulation of natural convection is not easy, thus the students had to apply all their acquired knowledge to set up a simulation. This especially included the theoretical background on numerical methods.

After successfully setting up the simulations, the students utilized the results to find criteria for natural convection. Further they compared the observations with the values of certain dimensionless numbers as e.g. the Rayleigh number and thus related their experiments to theoretical descriptions. The obtained results allowed a more detailed evaluation of the different possible flow regimes. As result a map of the flow regimes was set up and compared to experimental data as e.g. in [15, 16] (analyze and evaluate).

3.4 Project Work

Both lectures closed by project work, which represents the highest level and includes all categories of the revised Bloom's Taxonomy (Fig. 2). The following subsections contain details on the formal setting and the grading scheme.

1) Setting: The project work started in the 13th week of the semester. The duration was flexible, at least three weeks but it was allowed to work in the semester breaks. During the scheduled lecture hours supervising was offered. Further it was possible to use the computers also outside the lecture hours. The students build small groups of three to five persons. The lecturers provided four topics, but the students were encouraged to create their own topic, as extra motivation for the project. The work starts with setting up a reasonable working plan, after a short familiarization phase.

2) Grading: To grade the students and find fair marks, a rubric shown in Fig. 3 was utilized. This choice was inspired by a talk of A. Glahe (Technical University of Darmstadt, Germany) [17]. More details can e.g. be found in [18].

The use of rubrics has several advantages: First the grading becomes more objective. It is more consistent and transparent for the students, because it was available for them. Before starting work it is clear, which points are important. Further it simplifies and speeds up the ranking and in principle each assistant can contribute.

It was differed between five categories (Fig. 3). Each category consisted of several subcategories, where some were evaluated individually (S, in gray) and some were related to group work (G, white).

In each subcategory the students could receive from zero to four points. These points were weighted. At the end the sum of all weighted points defined an order, which made the students' performances comparable. The point criteria were defined in an extra table (Fig. 4), but they were not provided to the students.

At the end of the project, the students presented their work in a short 30 minutes talk on the obtained results. In this talk, each member of the group had to contribute. Finally an oral exam of 15-20 min was held with each group member separately.

Fig. 3 shows that the first half of the oral exam consisted of self-reflection. First the students were asked to judge their project results and point out open tasks. The next point was about group work and the pros and cons. This is one possible way to investigate the development of soft skills during project work. The authors emphasize, that negative answers concerning both points did not affect the final mark. In contrast, they show that the students were able to do a reasonable self-reflection. The last part of the oral exam was concerned with technical questions around the project and the lectures, to check the acquired knowledge during the course.

4. Example Projects in COMSOL

4.1 Example for "Simulation Methods"

This following project was a students' choice and not a predefined topic: "Simulation of Wing-tip Vortices". The group consists of five members from five different countries.

The first part of their work consists on investigating the principle of wingtips (winglets). The main advantage is to have smaller vertices and less drag in the case of small velocities. As a result the fuel consumption is reduced.

Then, before simulating the effect of wingtips, they had to create the geometries. Actually it turns out, that this was one of the most complicated parts of the project. Their work around was to set up a Matlab program, which parameterizes the wing profile (NACA 2415). So they were able to create the geometries with the simple geometry tools, provided by COMSOL.

After that the students implemented several simulations, concerning different designs of wingtips (vertical version, incline version, wing fence design etc.). First they started with a stationary solution and then they considered transient cases, which results in movies that clearly show how vertices develop and that wingtips reduce them. One special difficulty in the simulations was to find reasonable boundary and initial conditions.

Area	Weight	Weigth	Criteria	Rating (0-4)	weighted rating	Group/ single rating (G,S)
Project Content	0,2					
		0,08	Creativity/Own Ideas	4	0,32	G
		0,12	Application of learnd know-how	4	0,48	G
Quality of results	0,1					
		0,04	Reached project goal/milestones	4	0,16	G
		0,06	Reasonable Results	4	0,24	G
Way of working	0,2					
		0,1	Reasonable, methodic approach	4	0,4	G
		0,04	Project plan/ time management	4	0,16	G
		0,03	Independent working	4	0,12	S
		0,03	Motivation	4	0,12	S
Final presentation	0,2			4		
		0,055	Volume and content	4	0,22	G
		0,06	Structure of talk	4	0,24	G
		0,055	Time-limit	4	0,22	G
		0,01	Speaks clearly/ presentation skills	4	0,04	S
		0,01	Vocabluary (technical terms)	4	0,04	S
		0,01	Discussion	4	0,04	S
Oral exam	0,3					
		0,05	Selfreflexion of results	4	0,2	S
		0,05	Selfrefl. of working process/group work	4	0,2	S
		0,1	Theoretical background	4	0,4	S
		0,1	Knowledge on project topics	4	0,4	S
			Sum of weighted rating		4	
			Maximal sum of weighted ratings		4	

Figure 3: Rubric for grading the project work

Points	0	2	4
Creativity/Own Ideas	no independent working, all necessary steps and ideas were given by the supervisor	The project would have been a failure, without supervising and providing the important hints and ideas. Nonetheless the students had some own ideas and smaller parts were done independently.	All ideas and steps were done independently. Further the students had interesting new ideas, which were not expected at the biginning of the project.

Figure 4: Criteria for assigning the points in the category: creativity/own ideas

At the end of the project, the students were able to show qualitatively with their simulations that wingtips reduce vertices. Further they observed small differences between the different wingtip types (compare Fig. 5).
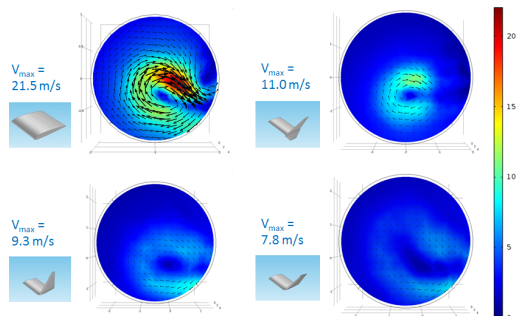

Figure 5: Result of students' project: Comparison of different wingtip designs.

Nonetheless due to the missing data on real wingtips and due to the unknown exact initial conditions, unfortunately their results are not adequate enough to judge, which wingtip design is the best. The students pointed out, that in order to reach this goal, more information and experiments for validation are necessary, which is far beyond the scope of such a small project.

4.2 Example for "Simulation and Optimization in Virtual Engineering"

This is a project given to the students as first project before they work out the final project on their own. In a drying process from car painting, the car drives through an oven. On the oven wall there are several infrared lamps installed. The infrared radiation heats up the car body and drys the painting. In addition there is air flowing from inlets on the bottom to an outlet on the top of the oven (Fig. 6).
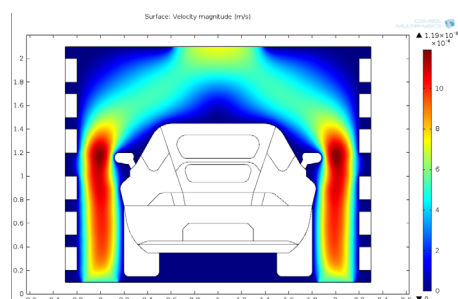

Figure 6: Simplified 2D situation and velocity magnitude (m/s)

The students should set up an optimization problem with the following two goals: Controlling the lamp power in order to achieve a uniform temperature at the car body and in addition saving energy. To achieve this, they should use the LiveLink to MATLAB.

The geometry file and the parameters are given in a short description, including the first steps for the optimization process.

5. Observations and Evaluation

This section presents the evaluation of the courses. First from the students' point of view and then the lecturers' observations.

5.1 Students' Opinions

Through the Universities evaluation and additional questioning from the professor, the opinion from the student were worked out and summarized for both courses. The students liked the mix of theory and practice. Through the final project work they clearly recognized their our succes in the study. During the project they learned some new aspects, which were not tought in the lectures.

For improvemend they would like to have more explanations and discussions on the physical and mathematical backround , as well as more informations about connection with other software, e.g. CAD tools.

5.2 Lecturers' Observations

Basically the authors agree with the students, especially that the mixture of theory and practice was a success. Certainly the students would like to have a lecture, which covers all relevant topics, but the course content is a good compromise. The observations show, that the students worked harder and were very motivated, compared to common exams.

Through their good organization during the projects, they showed clearly, that project work trains "soft skills". This accords with [12, 13].

Altogether the course was successful and all goals were attained. Further the lecturers' and the students' observations concerning the study success and the effect of project work accord to results achieved in other papers.

Certainly the learning success strongly depends on the number of students and on the

individual persons. Thus a small class of nine to 15 students facilitates teaching. Nonetheless the authors believe that it is possible to transfer the basic ideas and approaches to larger classes of up to 20-30 students. In this case more regulating by the lecturers, especially in the computer exercises, is necessary. For larger courses one either has to split the group for the computer exercises or has to employ some assistants.

In future, improvements by E-Learning are planned, where the software handling is shown in short videos. Further by exchanging examples from COMSOL's model library with industrial cases, teaching will be closely related to ongoing research. In addition it is planned to give the course as block courses.

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