

Modeling Scheil Cooling – Thermodynamic and Multiphysics Simulation

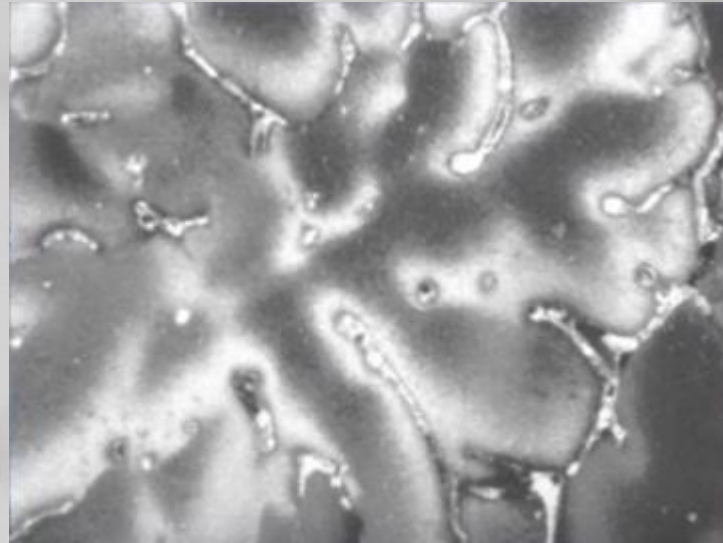
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M4dynamics

Multicomponent Solidification

The Problem:

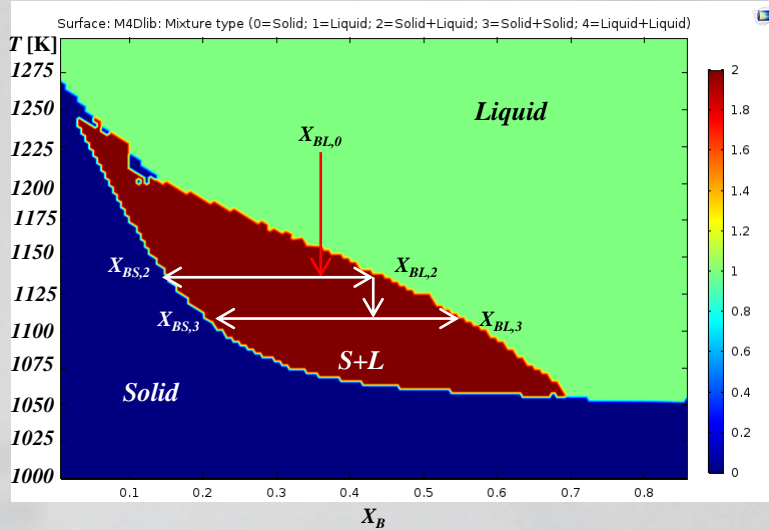
- Process depends on both Temperature and composition
- Most system are highly non-ideal (metal alloys, mattes, slags)
- During real solidification, solid doesn't equilibrate with residual liquid → non uniform solid concentration



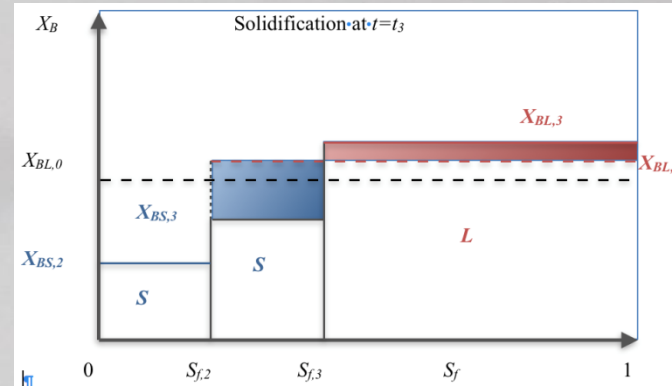
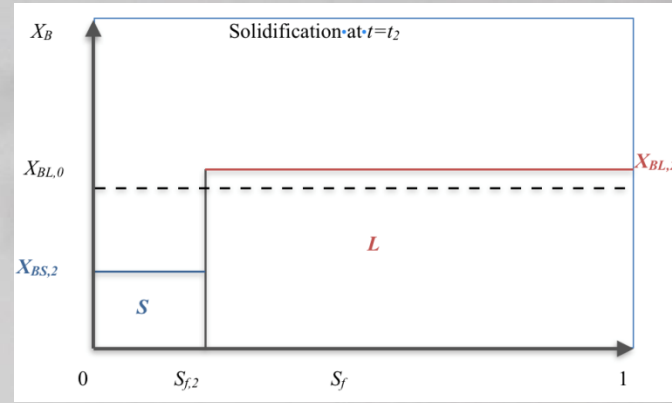
Coring in Al cast. Picture from Prof. David Dye, Imperial College, MSE104 course notes, 2012.

Multicomponent Solidification

- Rapid cooling Concept



Phase diagram for binary alloy, calculated in COMSOL using M4Dlib



Implementation in COMSOL

- The real solidification model was implemented using 0D ODE module → the same concept can later be applied to 1,2,3D
- Three variables were defined:

- T : Temperature $\frac{\partial T}{\partial t} = R$

- S : Solid fraction $S = S_f(T, X)$ ← from M4Dlib

- XB : Composition of component B $\frac{\partial X_{B,L}}{\partial t} = R_{B,L}$

Source Term: “enrichment”

- Balance from solidification from t_2 to t_3 :

$$(S_{f,3} - S_{f,2})(X_{BL,2} - X_{BS,3}) = (X_{BL,3} - X_{BL,2})(1 - S_{f,3})$$



$$\frac{\partial X_{BL}}{\partial t} = \frac{(X_{BL} - X_{BS})}{L_f} \frac{\partial S_f}{\partial t}$$

Scheil Equations:

- Balance from solidification from t_2 to t_3 :

$$(S_{f,3} - S_{f,2})(X_{BL,2} - X_{BS,3}) = (X_{BL,3} - X_{BL,2})(1 - S_{f,3})$$

$$K_{BS,L} = \frac{X_{BS}}{X_{BL}} \rightarrow \downarrow$$

$$(1 - K_{BS,L}) \int_0^{S_f} \frac{dS_f}{(1 - S_f)} = \int_{X_{B0}}^{X_{BL}} \frac{dX_{BL}}{X_{BL}}$$



$$X_{BL} = X_{B0} (1 - S_f)^{(K_{BS,L} - 1)}$$

$$X_{BS} = K_{BS,L} \cdot X_{B0} (1 - S_f)^{(K_{BS,L} - 1)}$$

Results – Scheil Cooling

- Initial conditions:

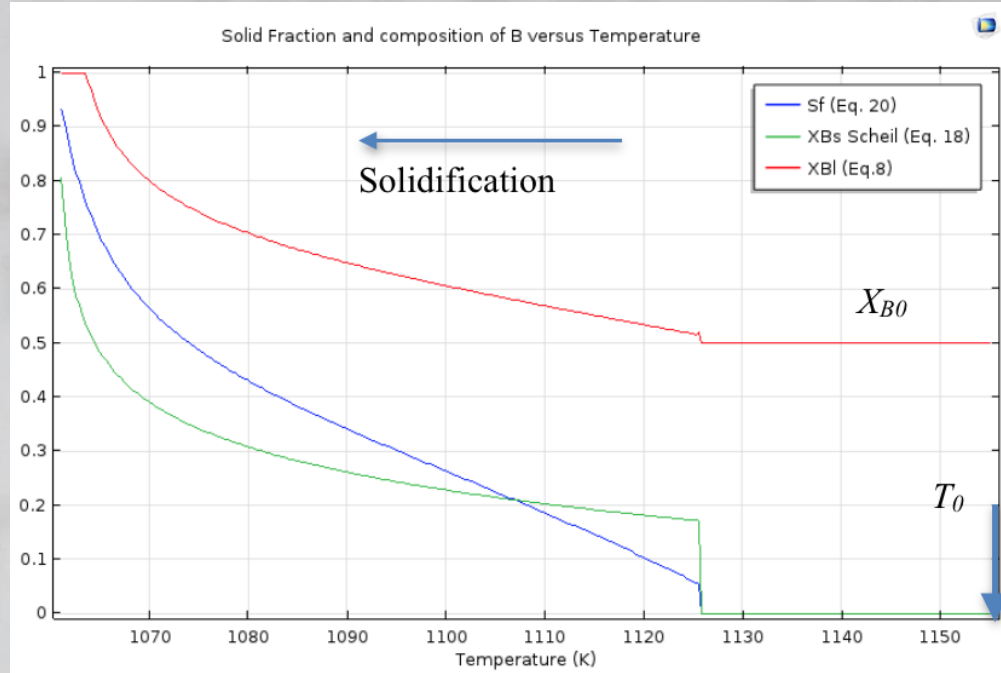
$$T_0 = 1155\text{K}$$

$$X_{B0} = 0.5$$

- $T_{liquidus} = 1127\text{K}$

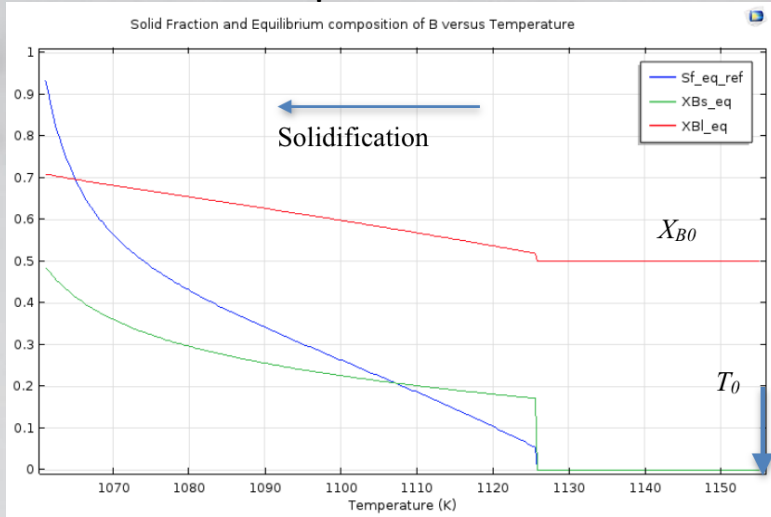
- Final solid comp.:

$$X_{BS} = 0.8 (> 0.5)$$



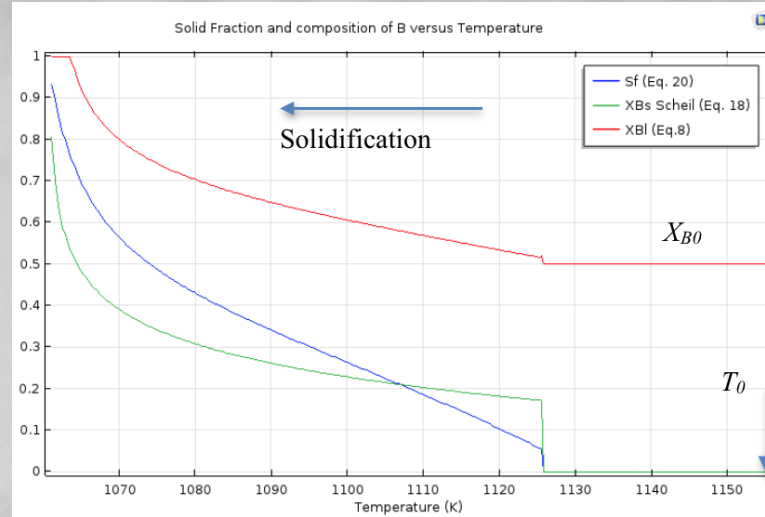
Scheil vs Equilibrium Solidification

Equilibrium



- $T_{solidus} = 1061\text{K}$
- Final solid comp.:
 $X_{BS} = 0.5$

Scheil



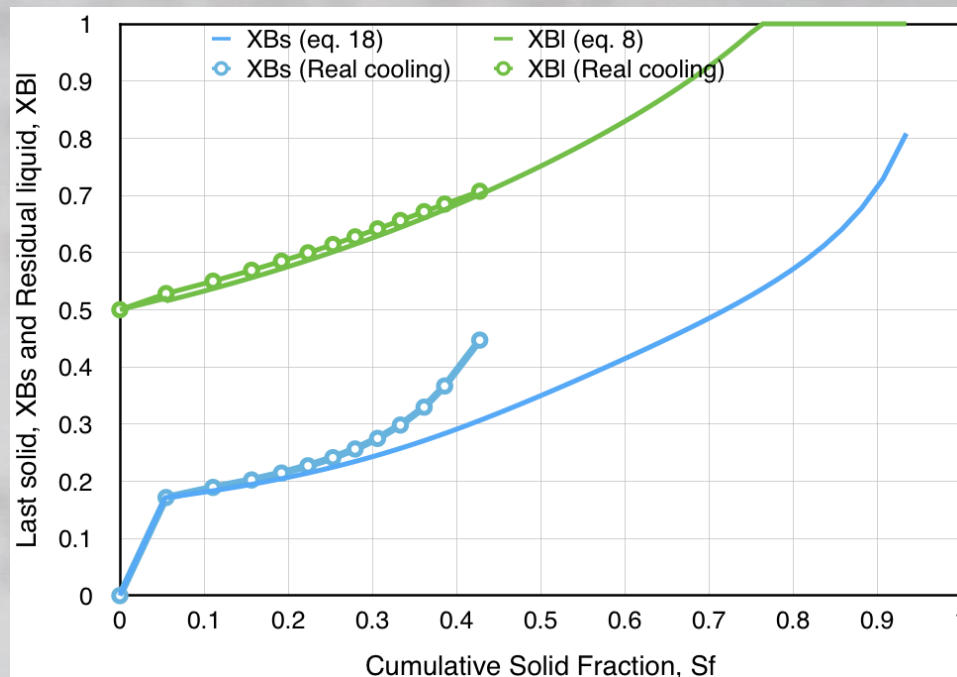
- $T_{solidus} = 1061\text{K}$
- Final solid comp.:
 $X_{BS} = 0.8 (> 0.5)$

Scheil vs Real Solidification

Scheil approx. assumes constant partition: $K_{BS,L}$.
Instead:

$$-\ln(1 - S_f) = \int_{X_{B0}}^{X_{BL}} \frac{dX_{BL}}{X_{BL}(1 - K_{BS,L})}$$

- Real solidification:
 - $T_{solidus} = 1055K$



Conclusions

- Scheil Cooling has been implemented using COMSOL Multiphysics and M4Dlib
- The model has been solved using a 0D ODE model for three variables: T , S_f and X_B .
- A source term describing the “enrichment” of liquid was derived to calculate X_B based on thermodynamic functions.
- There are still some limitations to this model that has been solved in M4Dlib but yet to be implemented in external library for COMSOL:
 - $L_f \rightarrow 0$
 - Constant partition K

