COMSOL Implementation of a Multiphase Fluid Flow Model in Porous Media

M.A. Diaz-Viera[*,1], D.A. Lopez-Falcon[1], A. Moctezuma-Berthier[1], A. Ortiz-Tapia[1]
[1]Instituto Mexicano del Petroleo
*Corresponding author: Eje Central Lazaro Cardenas # 152, 07730, Mexico, D. F., mdiazv@imp.mx

Abstract: Multiphase fluid flow models in porous media are frequent in many important applications of Earth Sciences, such as in groundwater contamination, subsurface remediation and reservoir modeling. The aim of the present work is to implement in COMSOL Multiphysics a multiphase fluid flow model in porous media, also known in the oil reservoir engineering literature as a black oil model, using a standard finite element approach. In particular, we are interested to apply this model coupled with a multiphase, multicomponent transport model to study Enhanced Oil Recovery processes at laboratory scale. The model is based on the oil phase pressure and total velocity formulation in which the capillary pressure is taken in account. The numerical implementation is validated comparing the results with the analytical solution from the Buckley-Leverett theory, and it is shown its performance for a water flooding case study through a sandstone core.

Keywords: black oil, multiphase fluid flow, porous media, capillary pressure model, EOR.

1. Introduction

The goal of this work is to implement a multiphase fluid flow model in porous media. In particular, we are interested to apply this model coupled with a multiphase, multicomponent transport model, to study Enhaced Oil Recovery processes at laboratory scale.

In view of the scale and resolution requirements for the flow model, it could be acceptable to perform the implementation making use of the standard finite element framework provided in COMSOL Multiphysics.

The model is based on the oil phase pressure and total velocity formulation given by Chen Z. [1] in which the capillary pressure, i.e., pressure difference between phases, is taken in account.

The implementation is validated comparing the numerical results with the analytical solution for the Buckley-Leverett problem, and it is shown its performance for a water flooding case study through a sandstone core (reservoir rock sample).

2. Multiphase Fluid Flow Model in Porous Media

2.1 The Black Oil Model

A multiphase fluid flow model through porous media, also known in the reservoir engineering literature as black oil or beta model [2], can be obtained applying the systematic approach for modeling continuum systems, which basically consists of deriving a set of local balance equations for intensive properties (in this case mass) by components and phases included in the model [3].

The main assumptions considered in the model are: (1) there are three phases: water (w), oil (o) and gas (g), (2) porous matrix and fluids are slightly compressible, (3) oil phase consists of two components: non volatile oil and dissolved gas, while the water and gas phases are pure, i.e., they are compound for only one component respectively, (4) diffusion will be neglected for all phases, (5) it is considered that the porous medium is fully saturated, but the phases are separated in the pore space and (6) the whole system is in local thermodynamic equilibrium.

The equation system of the black oil model [2] is given by

$$\phi \frac{\partial}{\partial t}\left(\frac{S_w}{B_w}\right) + \nabla \cdot \left(\frac{1}{B_w}\underline{u}_w\right) = q_w \qquad (1)$$

$$\phi \frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right) + \nabla \cdot \left(\frac{1}{B_o}\underline{u}_o\right) = q_o \qquad (2)$$

$$\phi \frac{\partial}{\partial t}\left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o}\right) + \nabla \cdot \left(\frac{1}{B_g}\underline{u}_g + \frac{R_{so}}{B_o}\underline{u}_o\right) = q_g \quad (3)$$

Here, $\underline{u}_\alpha$ represents the volumetric phase velocity, which can be expressed by the Darcy law as follows

$$\underline{u}_\alpha = -\frac{k k_{r\alpha}}{\mu_\alpha}\left(\nabla p_\alpha - \rho_\alpha \tilde{\underline{g}}\right); \quad \alpha = g,o,w \qquad (4)$$

Where ϕ and $\underline{k}$ denote the porosity and absolute permeability tensor of the porous system, while S_α, μ_α, ρ_α, p_α, $\underline{u}_\alpha$, B_α, $k_{r\alpha}$ and q_α are the saturation, viscosity, density, pressure, Darcy velocity, formation volume factor, relative permeability, and external source term, for each phase $\alpha = g,o,w$, respectively. While R_{so} is the gas solubility and $\tilde{\underline{g}}$ is the gravitational, downward-pointing, constant vector.

As can be observed the equation system (1)-(3) consists of only three equations and it contains six unknowns $\left(p_o, p_g, p_w, S_o, S_g, S_w\right)$, therefore the following three additional equations are required for the system to be determined:

$$S_g + S_o + S_w = 1$$
$$p_{cgo}\left(S_g\right) = p_g - p_o \qquad (5)$$
$$p_{cow}\left(S_w\right) = p_o - p_w$$

where p_{cgo} and p_{cow} are gas-oil and oil-water capillary pressures, respectively.

2.2 Formulation based on Oil Pressure and Total Velocity

Usually the black oil model presented above is solved numerically by finite difference or finite volume methods [4]. For numerical implementation using a finite element method the black oil model has to be reformulated in more convenient manner. A review of such kind of formulations is given in [1]. Here, we will present a particular formulation based on oil phase pressure and total velocity.

For convenience, the following notation is introduced: for $\alpha = w,o,g$, $\lambda_\alpha = k_{r\alpha}/\mu_\alpha$ are the phase mobility functions, $\lambda = \sum \lambda_\alpha$ is the total mobility and $f_\alpha = \lambda_\alpha/\lambda$ are the fractional flow functions. So that $\sum f_\alpha = 1$. The total velocity is defined as $\underline{u} = \sum \underline{u}_\alpha$.

Since the oil is a continuous phase and consequently its pressure is well behaved, we are going to define the oil phase pressure p_o as the pressure variable p.

After performing the corresponding notation substitution and appropriate manipulation on the black oil model (1)-(3), the system of equations transforms in a single pressure equation

$$\nabla \cdot \underline{u} = \sum_{\beta=g,w,o} B_\beta \left(q_\beta - \phi S_\beta \frac{\partial}{\partial t}\left(\frac{1}{B_\beta}\right) - \underline{u}_\beta \cdot \nabla\left(\frac{1}{B_\beta}\right)\right)$$
$$-B_g \left(R_{so}q_o + \frac{\phi S_o}{B_o}\frac{\partial R_{so}}{\partial t} + \frac{1}{B_o}\underline{u}_o \cdot \nabla R_{so}\right) \qquad (6)$$

and two saturation equations, for $\alpha = w,o$

$$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \underline{u}_\alpha$$
$$= B_\alpha\left(q_\alpha - \phi S_\alpha \frac{\partial}{\partial t}\left(\frac{1}{B_\alpha}\right) - \underline{u}_\alpha \cdot \nabla\left(\frac{1}{B_\alpha}\right)\right); \qquad (7)$$

where, total and phase velocities are given by

$$\underline{u} = -\underline{k}\lambda\left(\nabla p_o - f_w\nabla p_{cow} + f_g\nabla p_{cgo} - \tilde{\underline{g}}\nabla z \sum_\beta f_\beta \rho_\beta\right);$$
$$\underline{u}_w = f_w\underline{u} + \underline{k}f_w\lambda_o\left(\nabla p_{cow} - (\rho_o - \rho_w)\tilde{\underline{g}}\nabla z\right)$$
$$+\underline{k}f_w\lambda_g\left(\nabla\left(p_{cgo} + p_{cow}\right) - (\rho_g - \rho_w)\tilde{\underline{g}}\nabla z\right);$$
$$\underline{u}_o = f_o\underline{u} + \underline{k}f_o\lambda_w\left(-\nabla p_{cow} + (\rho_o - \rho_w)\tilde{\underline{g}}\nabla z\right)$$
$$+\underline{k}f_o\lambda_g\left(\nabla p_{cgo} - (\rho_g - \rho_o)\tilde{\underline{g}}\nabla z\right);$$
$$\underline{u}_g = f_g\underline{u} + \underline{k}f_g\lambda_w\left(-\nabla\left(p_{cow} + p_{cgo}\right) - (\rho_w - \rho_g)\tilde{\underline{g}}\nabla z\right)$$
$$+\underline{k}f_g\lambda_o\left(-\nabla p_{cgo} - (\rho_o - \rho_g)\tilde{\underline{g}}\nabla z\right); \qquad (8)$$

2.3 Laboratory Scale Flow Model

The system of equations given in (6)-(8) represent a quite general multiphase fluid flow model in porous media, but as was mentioned before, we are interested to apply our flow model to reproduce the flow behavior at laboratory scale. Hence, taking into account the laboratory conditions, we can make the following further assumptions to the flow model:

- Due to the small variation range of the pressure, water and oil phases can be considered incompressible, i.e., $B_o = B_w = 1$.

- The term $\underline{u}_g \cdot \nabla\left(1/B_g\right)$ can be neglected since the pressure gradient is small ($\nabla p \ll 1$).

- The gas dissolution in the oil phase doesn't take place ($R_{so} = 0$).
- The effect of gravity $\tilde{g}$ can be ignored.
- The porous medium is homogeneous ($\phi = const.$) and isotropic ($\underline{\underline{k}} = k\underline{\underline{I}}$).

After including the previous simplifications in the equation system (6)-(8) we obtain:

$$-\phi\left(1 - S_o - S_w\right)\frac{1}{B_g}\frac{dB_g}{dp}\frac{\partial p_o}{\partial t} + \nabla \cdot \underline{u} \qquad (9)$$

$$= q_o + q_w + B_g q_g$$

$$\phi\frac{\partial S_o}{\partial t} + \nabla \cdot \underline{u}_o = q_o;$$

$$\phi\frac{\partial S_w}{\partial t} + \nabla \cdot \underline{u}_w = q_w; \qquad (10)$$

Where, total and phase velocities can be rewritten as follows

$$\underline{u} = -k\lambda\nabla p_o + k\lambda f_w\nabla p_{cow} - k\lambda f_g\nabla p_{cgo};$$

$$\underline{u}_w = f_w\underline{u} + kf_w\left(\lambda_o + \lambda_g\right)\nabla p_{cow} + kf_w\lambda_g\nabla p_{cgo};$$

$$\underline{u}_o = f_o\underline{u} - kf_o\lambda_w\nabla p_{cow} + kf_o\lambda_g\nabla p_{cgo}; \qquad (11)$$

$$\underline{u}_g = f_g\underline{u} - kf_g\lambda_w\nabla p_{cow} - kf_g\left(\lambda_o + \lambda_w\right)\nabla p_{cgo};$$

Finally, replacing velocities in pressure and saturation equations (9)-(10) by the expressions given in (11) and expressing the capillary pressure gradients in terms of saturation:

$$\nabla p_{cow} = \frac{dp_{cow}}{dS_w}\nabla S_w; \quad \nabla p_{cgo} = \frac{dp_{cgo}}{dS_g}\nabla S_g; \qquad (12)$$

the equation system for the fluid flow model in porous media becomes:

Pressure equation

$$\phi\left(1 - S_o - S_w\right)c_g\frac{\partial p_o}{\partial t}$$

$$-\nabla \cdot \left\{k\lambda\nabla p_o - k\lambda\left(f_w\frac{dp_{cow}}{dS_w} + f_g\frac{dp_{cgo}}{dS_g}\right)\nabla S_w\right\} \quad (13)$$

$$-\nabla \cdot \left\{-k\lambda f_g\frac{dp_{cgo}}{dS_g}\nabla S_o\right\} = q_o + q_w + B_g q_g$$

where $c_g = -\dfrac{1}{B_g}\dfrac{dB_g}{dp_o}$ is the gas compressibility.

Saturation equations

$$\phi\frac{\partial S_o}{\partial t} - \nabla \cdot \left(k\lambda f_o\nabla p\right) = q_o;$$

$$\phi\frac{\partial S_w}{\partial t} - \nabla \cdot \left(k\lambda f_w\nabla p - k\lambda f_w\frac{dp_{cow}}{dS_w}\nabla S_w\right) \qquad (14)$$

$$= q_w;$$

Total and phase velocities

$$\underline{u} = -k\lambda\nabla p_o + k\lambda f_w\frac{dp_{cow}}{dS_w}\nabla S_w$$

$$+ k\lambda f_g\frac{dp_{cgo}}{dS_g}\nabla S_w + k\lambda f_g\frac{dp_{cgo}}{dS_g}\nabla S_o;$$

$$\underline{u}_w = -k\lambda f_w\nabla p_o + k\lambda f_w\frac{dp_{cow}}{dS_w}\nabla S_w; \qquad (15)$$

$$\underline{u}_o = -k\lambda f_o\nabla p_o;$$

$$\underline{u}_g = -k\lambda f_g\nabla p_o + k\lambda f_g\frac{dp_{cgo}}{dS_g}\nabla\left(S_w + S_o\right)$$

3. Numerical Implementation

A brief review of the state of the art literature concerning the numerical implementation of multiphasic fluid flow model reflects that the finite difference (FD) and finite volume (FV) methods are the general framework for numerical simulation in very large problems [4]; however, the basic mixed finite element (MFE) method [5] has shown to be superior for accurate flux calculation in heterogeneous media in comparison to conventional FD and FV methods.

On the other hand, a mixed finite element approach requires a special Raviart-Thomas mixed space for base and weighting functions, which makes more difficult its implementation.

In view of the scale and resolution requirements for our flow model, we decided that it could be acceptable to perform the implementation making use of the standard finite element framework provided in COMSOL Multiphysics [6].

In particular, the numerical implementation of previously derived model was accomplished using the PDE mode for time dependent analysis in the coefficient form.

Although the model presented in the previous section is triphasic (water, oil and gas) hereafter, for simplicity and without lost of generality, we

will reduce our discussion about the numerical implementation to the biphasic case (water, oil).

For the biphasic case, i.e., we have only two phases: water (w) and oil (o). It is assumed that the fluids are incompressible, and consequently the equation system (13)-(15) is simplified as follows:

$$-\nabla \cdot (k\lambda \nabla p) + \nabla \cdot \left(k\lambda_w \frac{dp_{cow}}{dS_w} \nabla S_w \right)$$
$$= q_w + q_o; \tag{16}$$

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \left(k\lambda_w \frac{dp_{cow}}{dS_w} \nabla S_w \right)$$
$$-\nabla \cdot (k\lambda_w \nabla p) = q_w; \tag{17}$$

The system of equations (16)-(17) can be rewritten in matrix form as follows:

$$\begin{pmatrix} 0 & 0 \\ 0 & \phi \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} p \\ S_w \end{pmatrix}$$

$$+\nabla \cdot \left\{ -\begin{pmatrix} k\lambda & -k\lambda_w \dfrac{dp_c}{dS_w} \\ k\lambda_w & -k\lambda_w \dfrac{dp_c}{dS_w} \end{pmatrix} \nabla \begin{pmatrix} p \\ S_w \end{pmatrix} \right\} \tag{18}$$

$$= \begin{pmatrix} q_w + q_o \\ q_w \end{pmatrix}$$

The previous matrix representation can be translated in straightforward manner to the standard COMSOL notation in coefficient form:

$$\mathbf{u} \equiv \begin{pmatrix} p \\ S_w \end{pmatrix}, \mathrm{d}_a \equiv \begin{pmatrix} 0 & 0 \\ 0 & \phi \end{pmatrix},$$

$$c \equiv \begin{pmatrix} k\lambda & -k\lambda_w \dfrac{dp_{cow}}{dS_w} \\ k\lambda_w & -k\lambda_w \dfrac{dp_{cow}}{dS_w} \end{pmatrix}, \mathrm{f} \equiv \begin{pmatrix} q_w + q_o \\ q_w \end{pmatrix} \tag{19}$$

where $e_a, \alpha, \gamma, \beta, a \equiv 0$.

To complete the model only remains to define suitable constitutive laws for relative permeabilities (k_{rw}, k_{ro}) and oil-water capillary pressure p_{cow} and to prescribe proper initial and boundary conditions.

In the next two sections the flow model described above will be tested for two specific problems in 1-D. For defining initial and boundary conditions in those problems the following notations will be introduced:

Initial conditions

$$p(t_0) = p_0, \quad S_w(t_0) = S_{w0} \tag{20}$$

Boundary conditions

$$\mathrm{r}^{in} \equiv \begin{pmatrix} p^{in} \\ S_w^{in} \end{pmatrix}, \mathrm{g}^{in} \equiv \begin{pmatrix} g_p^{in} \\ g_{S_w}^{in} \end{pmatrix}$$

$$\mathrm{r}^{out} \equiv \begin{pmatrix} p^{out} \\ S_w^{out} \end{pmatrix}, \mathrm{g}^{out} \equiv \begin{pmatrix} g_p^{out} \\ g_{S_w}^{out} \end{pmatrix} \tag{21}$$

Where $q \equiv 0$, but h depends on the type of boundary conditions.

4. Numerical Simulations

4.1 Buckley-Leverett Problem

We first will verify the implemented numerical flow model with known analytical solutions. To this end, we solve the Buckely–Leverett problem in a homogenous medium with different fluid properties and zero capillary pressure [7].

We consider a 1-D horizontal homogeneous domain of length 300 m, initially saturated with oil. Water is injected with a constant flow rate at one end to displace oil to the other end, where the pressure is kept constant.

The relative permeability constitutive equations are given by:

$$k_{rw} = S_e^\omega; \quad k_{ro} = (1 - S_e)^\omega; \tag{22}$$

where $\omega = 1$ is for the linear case and $\omega = 2$ is for the quadratic case, whereas S_e is the effective or normalized saturation, which is defined as:

$$S_e = \frac{S_w - S_{rw}}{1 - S_{rw} - S_{ro}} \tag{23}$$

where S_{rw} and S_{ro} are the residual saturations for water and oil, respectively.

In relation to the general model description given in COMSOL notation in (19), only the matrix c and vector f are modified

$$c \equiv \begin{pmatrix} k\lambda & 0 \\ k\lambda_w & \varepsilon \end{pmatrix}; \quad f \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{24}$$

Since $p_{cow} \equiv 0$ and $q_w, q_o \equiv 0$. Note that it was introduced a small artificial diffusion coefficient (ε) in the saturation equation to stabilize the numerical solution, due to its hyperbolic nature, numerical instabilities can be appeared.

Table 1: Buckley-Leverett problem data

Property	Units	Value
Domain length (xmax)	m	300
Absolute Permeability (k)	m^2	1.00E-15
Porosity (ϕ)		0.2
Water viscosity (μ_w)	Pa.s	1.00E-03
Oil viscosity (μ_o)	Pa.s	1.00E-03
Residual water saturation (S_{rw})		0
Residual oil saturation (S_{ro})		0.2
Injection rate	m.s^{-1}	3.4722E-07
Production pressure	MPa	10
Artificial diffusion coefficient		1.00E-6

For this problem the initial conditions are
$$p(t_0) = p_0 \equiv 10\text{MPa}, \quad S_w(t_0) = S_{w0} \equiv 0 \quad (25)$$
and the corresponding boundary conditions are

$$r^{in} \equiv \begin{pmatrix} 0 \\ S_w^{in} \end{pmatrix}, g^{in} \equiv \begin{pmatrix} g_p^{in} \\ 0 \end{pmatrix}, h^{in} \equiv \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (26)$$

$$r^{out} \equiv \begin{pmatrix} p^{out} \\ 0 \end{pmatrix}, g^{out} \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix}, h^{out} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (27)$$

Where $S_w^{in} \equiv 0.8$, $g_p^{in} \equiv 3.47\text{E-7m} \cdot \text{s}^{-1}$,

$p^{out} \equiv 10\text{MPa}$.

The relevant data are taken from [8] and are provided in Table 1.

The simulations were carried out for three cases with different water-oil viscosity ratios combining two types of relative permeability models (linear and quadratic) for seven time periods, (see Table 2).

Table 2: Simulated cases for Buckley-Leverett problem

Cases	Relative permeability model (ω)	Viscosity ratio (μ_w/μ_o)	Simulation periods (t_{max}) [days]
(a)	1	2	300-900
(b)	1	2/3	300-900
(c)	2	2/3	300-900

4.2 Water Flooding Case Study

The second problem is about to reproduce the flow behavior in a water flooding experiment through a sandstone core under laboratory conditions. The intention is to couple this flow model with multiphase and multicomponent transport equations to study Enhaced Oil Recovery processes [9]. Data of this problem is given in Table 3.

In this case, the relative permeability constitutive equations are based on the Brooks-Corey model [10]:

$$k_{rw} = S_e^{\frac{2+3\theta}{\theta}}; \quad k_{ro} = (1-S_e)^2 \left(1 - S_e^{\frac{2+\theta}{\theta}}\right); \quad (28)$$

where θ characterizes the pore size distribution.

While oil-water capillary pressure is defined by the Leverett *J*-function:

$$p_{cow}(S_w) = p_t \left(\frac{S_w - S_{rw}}{1 - S_{rw} - S_{ro}} \right)^{(-1/\theta)} \quad (29)$$

where p_t is the entry or left threshold pressure assumed to be proportional to $(\phi/k)^{1/2}$.

Consequently, we can express the oil-water capillary pressure derivative as follows:

$$\frac{dp_{cow}}{dS_w}(S_w) =$$

$$-\frac{p_t}{\theta(1-S_{rw}-S_{ro})} \left(\frac{S_w - S_{rw}}{1 - S_{rw} - S_{ro}} \right)^{-(1+\theta)/\theta} \quad (30)$$

We impose the following initial conditions
$$p(t_0) = p_0 \equiv 10\text{MPa}, \quad S_w(t_0) = S_{w0} \equiv 0.2 \quad (31)$$
and boundary conditions

$$r^{in} \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix}, g^{in} \equiv \begin{pmatrix} g_p^{in} \\ 0 \end{pmatrix}, h^{in} \equiv \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad (32)$$

$$r^{out} \equiv \begin{pmatrix} p^{out} \\ 0 \end{pmatrix}, g^{out} \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix}, h^{out} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (33)$$

Where $g_p^{in} \equiv 5.3E - 07 \, m \cdot s^{-1}$, $p^{out} \equiv 10 MPa$.

Table 3: Water coreflooding experimental data

Property	Units	Value
Domain length (x_{max})	m	0.25
Absolute Permeability (k)	m^2	8.25E-13
Porosity (ϕ)		0.2
Water viscosity (μ_w)	Pa.s	1.00E-03
Oil viscosity (μ_o)	Pa.s	1.00E-02
Residual water saturation (S_{rw})		0.2
Residual oil saturation (S_{ro})		0.15
Injection rate	m.s^{-1}	5.3E-07
Production pressure	MPa	10
Brooks-Corey parameter (θ)		2
Entry threshold pressure (p_t)	MPa	1.00E-2

6. Results and Conclusions

In the figure 1, the numerical solutions of the Buckley–Leverett problem with linear relative permeabilities and viscosity ratio $\mu_w / \mu_o = 1$ for a time period of 300 days with different artificial diffusion coefficients are shown. We can observe that it is attained the best trade of in terms of efficiency and accuracy for an artificial diffusion coefficient value $\varepsilon = 1e - 7$.

Figures 2-4 show a quite well qualitative reproduction of the analytic solution behavior for the Buckley–Leverett problems for cases (a)-(c) described in table 2, respectively. These problems were numerically solved with the optimal artificial diffusion coefficient value $\varepsilon = 1e - 7$ previously obtained.

The numerical simulation of the water coreflooding experiment through a sandstone core during a time period of 24 hours is shown in figure 5. It can be observed the formation of a water front displacing the oil through the porous medium which is recovered at the production end of the core.

The main result of the present work is the implementation of a biphasic (water-oil) flow model in porous media, including capillary pressure, which coupled to multiphase and multicomponent transport equations could be useful to study Enhaced Oil Recovery processes at laboratory scale.

Even more, applying a flow model coupled with transport equations can be serve to study the impact in the flow conditions due to the porosity and permeability alterations by transport processes, such as adsorption of some fluent components.

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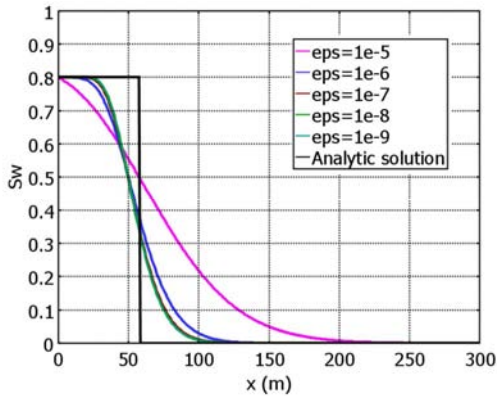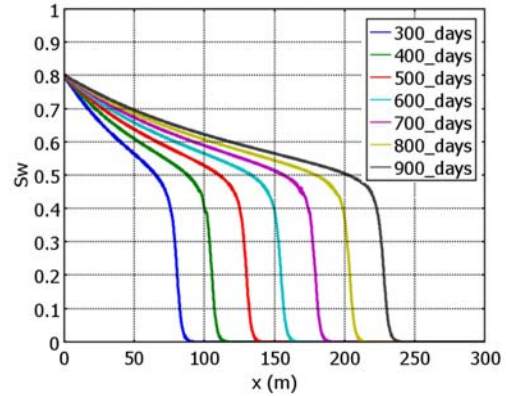
Figure 1. Numerical solutions of the Buckley–Leverett problem with linear relative permeabilities and viscosity ratio $\mu_w/\mu_o = 1$ for a period of 300 days, varying artificial diffusion coefficient (ε).
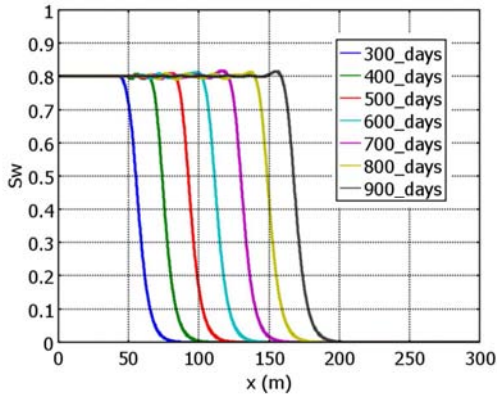

Figure 2. Numerical solutions of the Buckley–Leverett problem for case (a) with linear relative permeabilities and viscosity ratio $\mu_w/\mu_o = 2$ for time periods from 300 to 900 days.
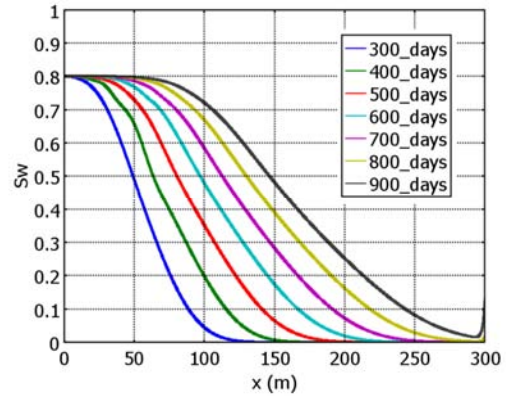
Figure 3. Numerical solutions of the Buckley–Leverett problem for case (b) with linear relative permeabilities and viscosity ratio $\mu_w/\mu_o = 2/3$ for time periods from 300 to 900 days.

Figure 4. Numerical solutions of the Buckley–Leverett problem for case (c) with quadratic relative permeabilities and viscosity ratio $\mu_w/\mu_o = 2/3$ for time periods from 300 to 900 days.
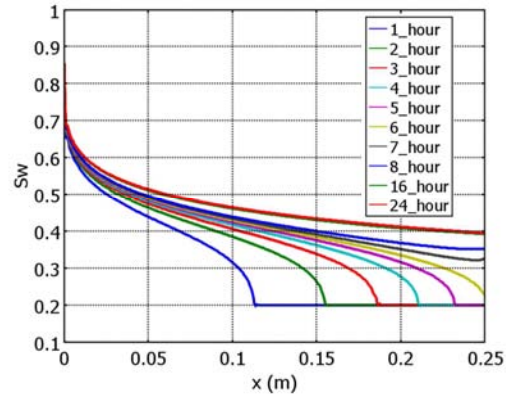
Figure 5. Numerical simulation of the water coreflooding experiment for a time period of 24 hours.