Presented at the COMSOL Conference 2008 Boston

Shape, Convection and Convergence

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Pryor Knowledge Systems, Inc.

COMSOL certified consultant

Who is Pryor Knowledge Systems?

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- **Located in Southeast Michigan**

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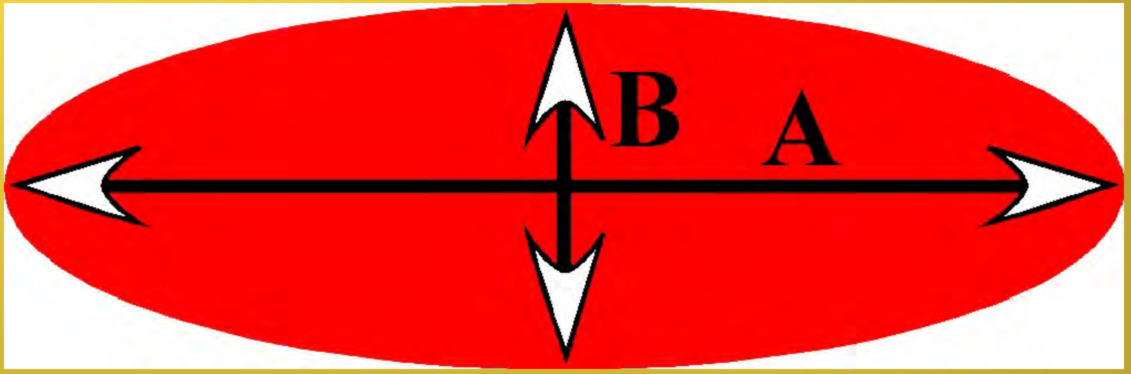
IV. What benefits are derived from applying this modeling technique?

Defining the Geometry of the Tank Top : What is the B/A Ratio?

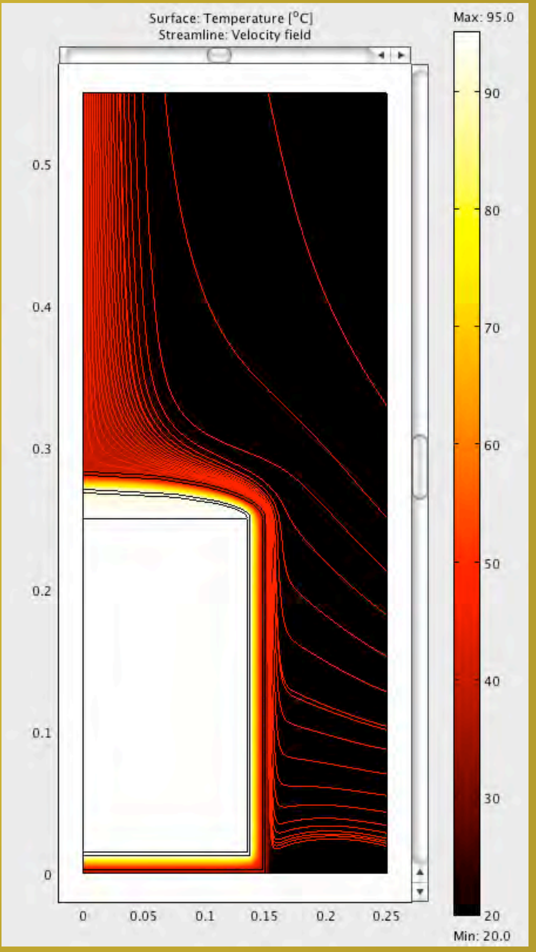


Geometric Specifications

Elliptical Tank Top : A-Semiaxis, B-Semiaxis

Introduction:

I. New Shape-Dependent Convection Model



Navier-Stokes Streamline Patterns generated by an Elliptically Shaped Tank Top

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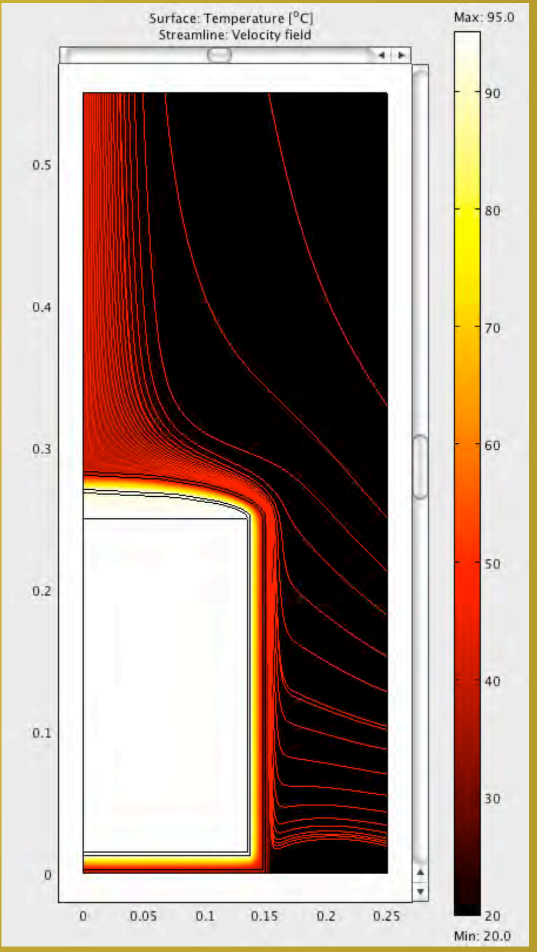

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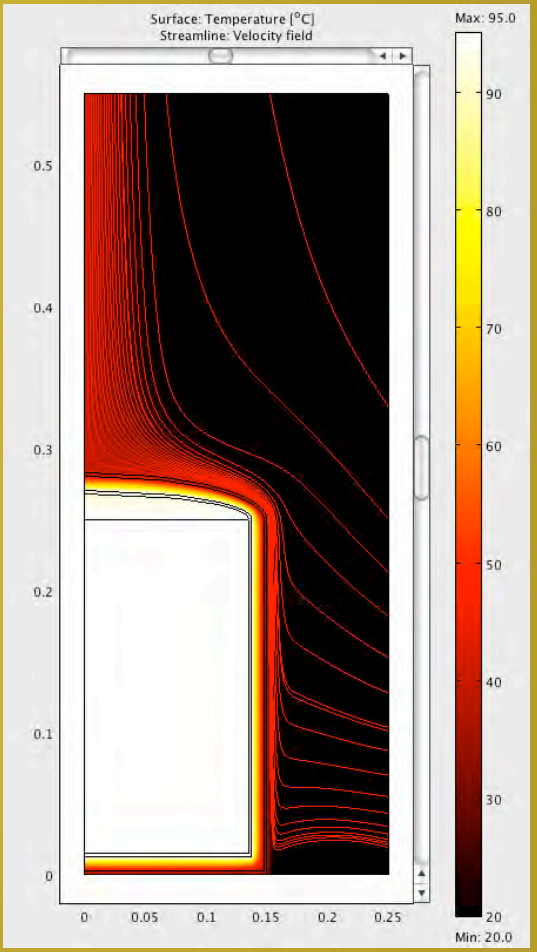

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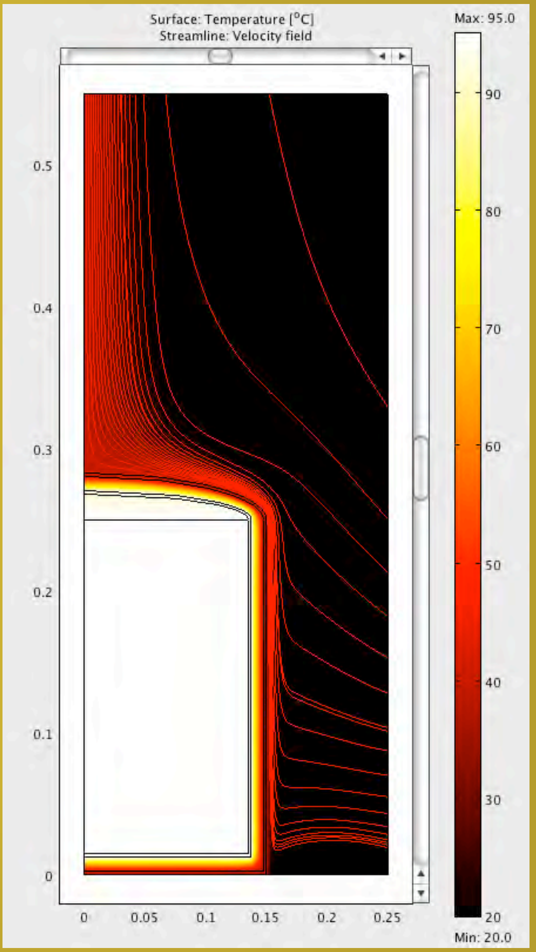

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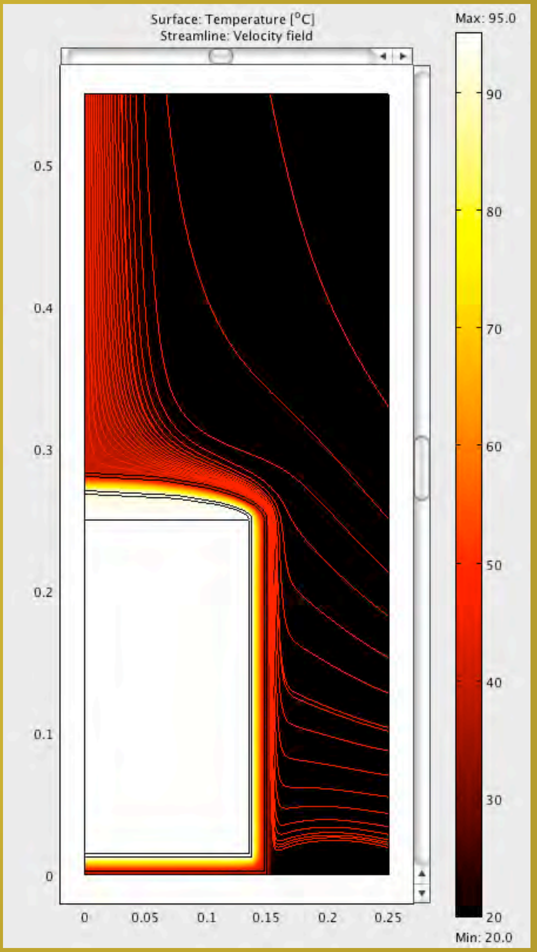
V. Shape-Dependent Model Convergence determined by both Tank Top Shape (B/A Ratio) and Modified Solver Parameters



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First Principles Applied to Fluid Flow

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III. **In this problem, the surface geometry (shape) of the top of the tank (B/A ratio) is the primary governing factor.**

The Physics:

I. Free Convection

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II. Driving Forces:

   a) temperature differential

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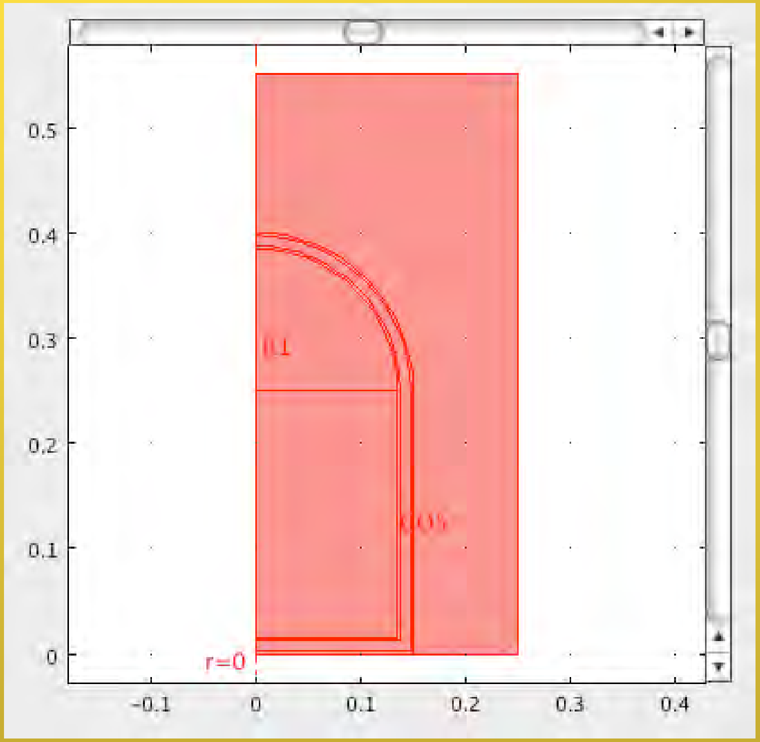
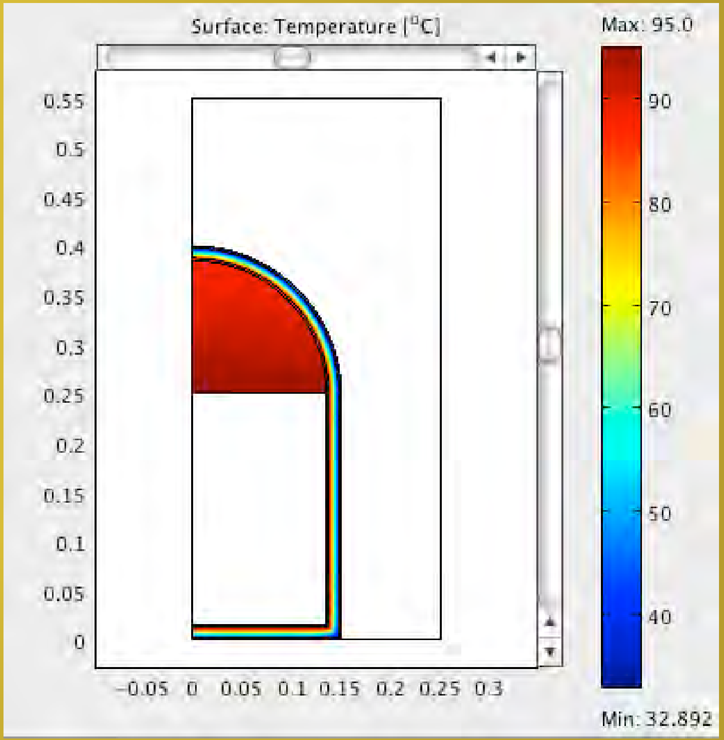
III. Tank (water) Temperature

COMSOL Heat Transfer Model 1:



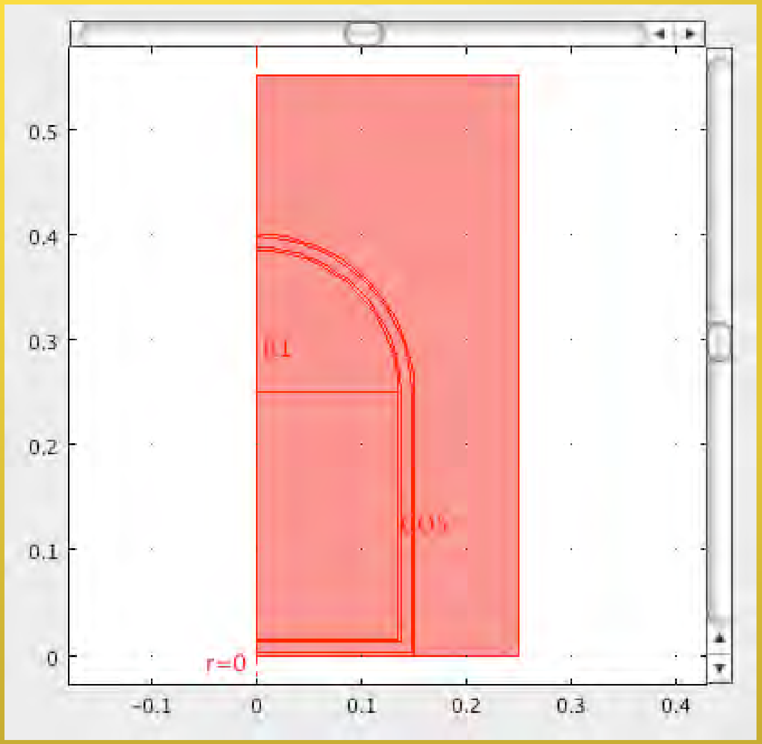


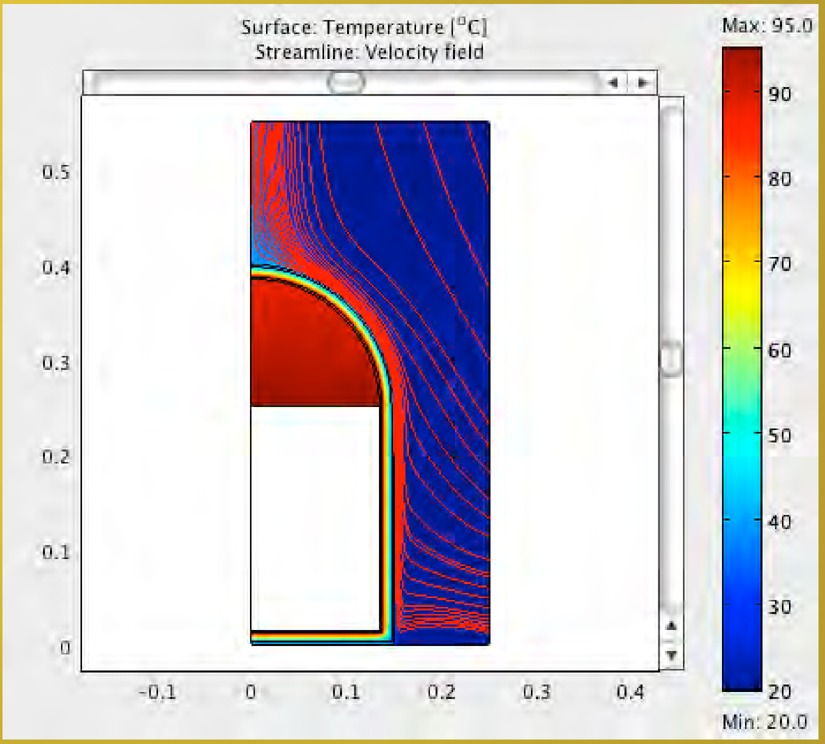
Tank Top B/A Ratio = 1.0

2D Axisymmetric Heat Transfer Initialization Model

Model 1 - Accurate Heat Transfer Results

COMSOL Navier-Stokes Model 2:





Tank Top B/A Ratio = 1.0

2D Axisymmetric Free Convection Model

Model 2 - Accurate Convection Results

Model Constants:

Name	Value	Description
k_insul	0.01[W/(m*K)]	k (Insulation)
p_atm	1[atm]	Air pressure
Length	0.4[m]	Height of tank*
T_water	95[degC]	Water Temperature
T_amb	20[degC]	Ambient Temperature
rho_insul	60[kg/m^3]	Density (Insulation)
Cp_insul	200[J/(kg*K)]	Heat Capacity (Insulation)
Length2	0.15[m]	Width of Tank

*This is the projected height and varies for different tank configurations: Length = B(Semi-axis length) + Height (Tank Rectangle)

Model Scalar Expressions:

Name	Expression
F_buoyancy*	**9.81[m/s^2]*(rho_air_ref-rho_chns)**
rho_air_ref**	**mat2_rho(p_atm[1/Pa],T_amb[1/K])[kg/m^3]**

*Buoyancy Force ** Density of Air @ STP

Setting up the Geometry for Models 1&2:

Tank Top Configuration (TTC) #	A[m]	B[m]	B/A	Convergence
1	0.15	0.15	1	Yes
2	0.15	0.20	1.33	Yes
3	0.15	0.05	0.33	Yes
4	0.15	0.03	0.20	HT: Yes N-S: No
5	0.15	0.04	0.267	Yes
6	0.15	0.033	0.22	Yes

Elliptical Tank Top Geometric Specifications

The Models 1&2 Geometry:

Models 1&2

B/A = 1

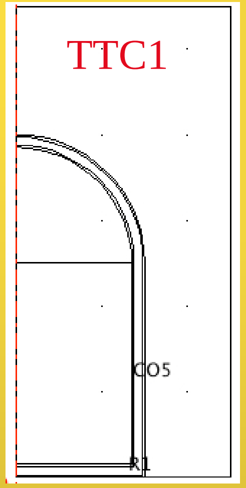
Length =

0.4[m]

Length2 =

0.15[m]

C = Yes



2D Axisymmetric Elliptical Tank Top Model Cross-Sections

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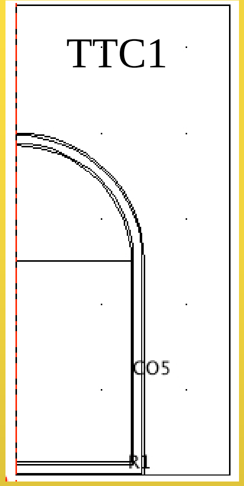

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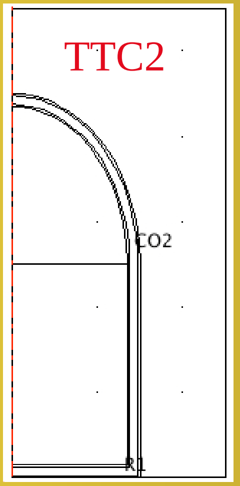

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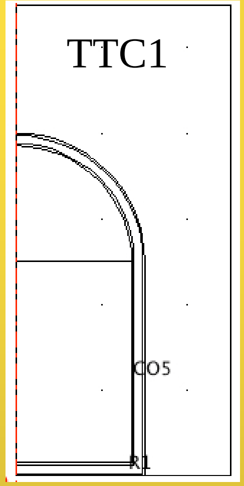

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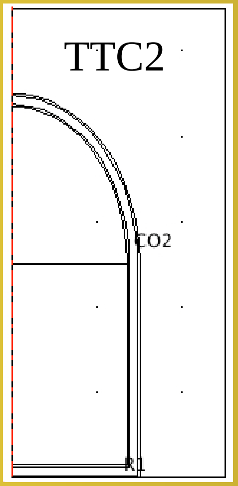

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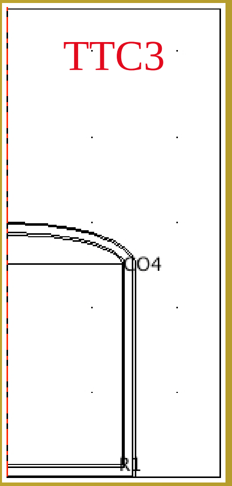

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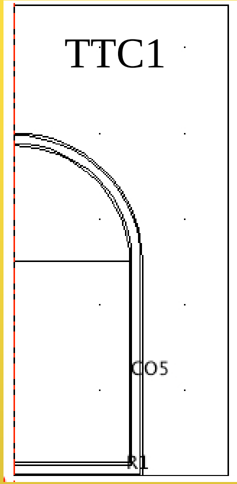

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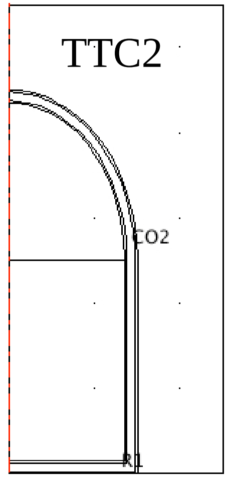

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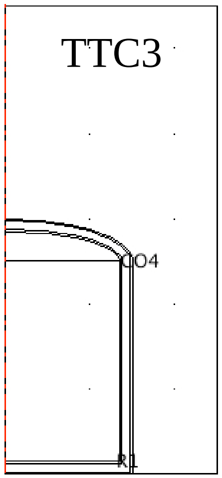
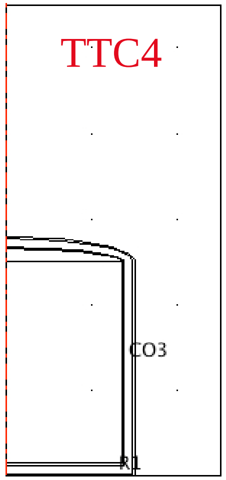


Models 1&2
B/A = 0.20
Length = 0.28[m]
Length2 = 0.15[m]
C = No



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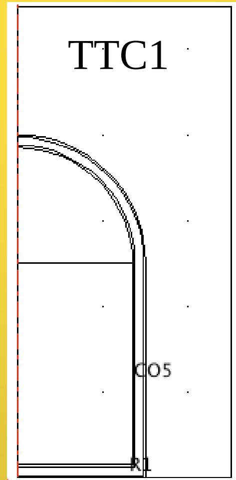

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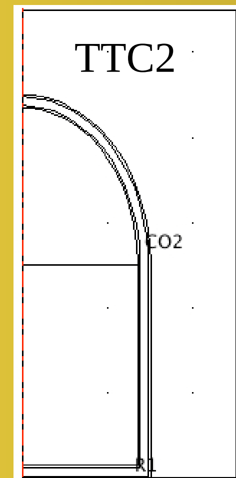

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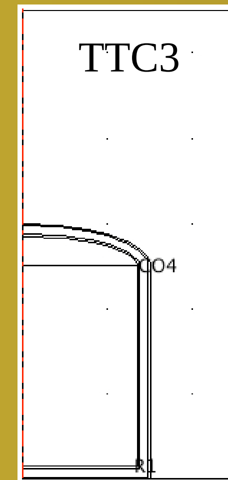

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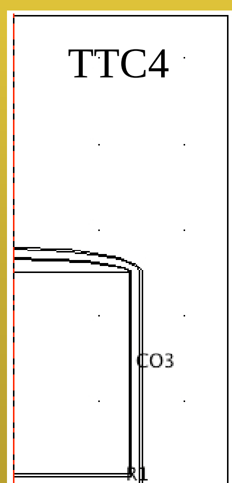
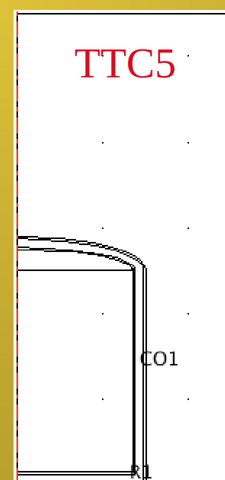


Models 1&2
B/A = 0.267
Length =
0.29[m]
Length2 =
0.15[m]
C = Yes



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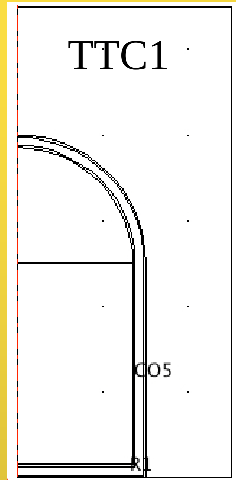

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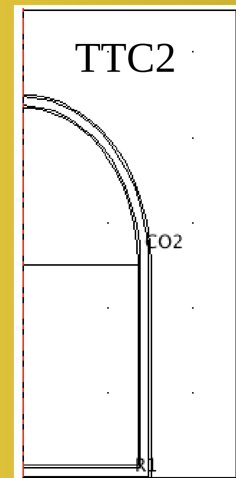

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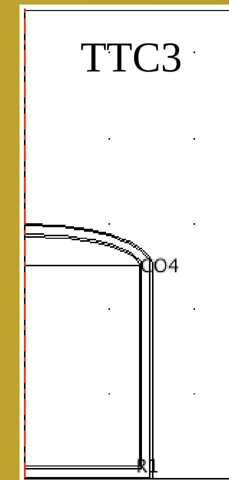

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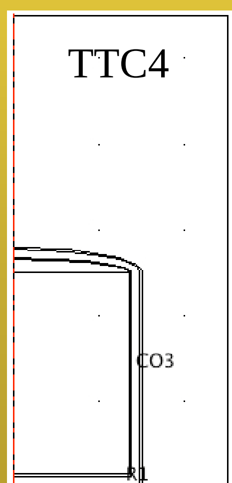

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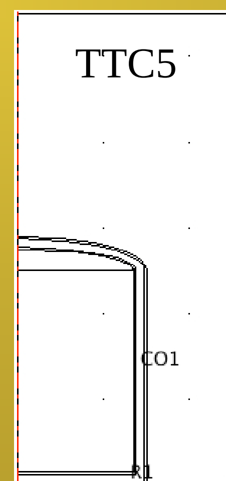


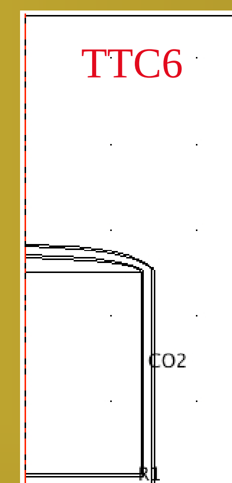
Models 1&2

B/A = 0.22

Length = 0.283[m]

Length2 = 0.15[m]

C = Yes



2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Setting up the Physics Subdomain Model 1(HT):

Subdomain #	Source	Material
1	BML**	structural steel
2	Constants	k_insul rho_insul Cp_insul
3	BML	structural steel
5	BML	Air*

**(BML) Basic Materials Library

*replace p with p_atm

Setting up the Physics Boundary Settings Model 1(HT):

Boundary	Condition	Setting
8, 14, 20	Temperature	T_water in T_0 edit window
23	Heat Flux	Load h: Nat. Vertical wall, L=height Change L_htgh to Length Enter T_amb in T_{inf} edit window
29	Heat Flux	Load h: Nat Horiz. plane, Upside, L=width Change L_htgh to Length2 Enter T_amb in T_{inf} edit window

Meshing and Solving Model 1(HT):

Use the default Mesh and Solver Parameters

Click Solve

Model 1(HT):Solutions

Model 1

B/A = 1

Length =

0.4[m]

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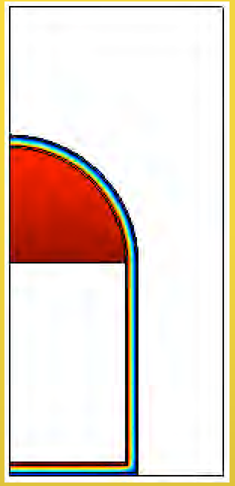

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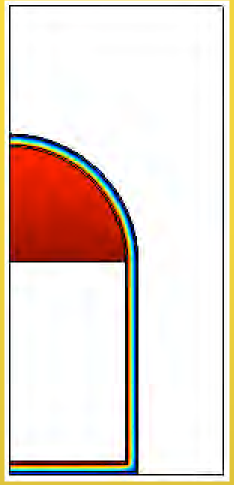

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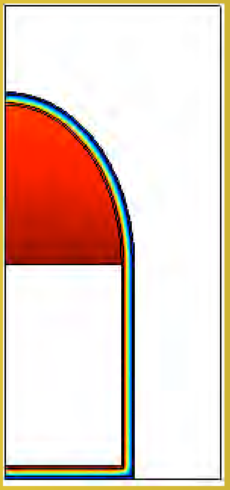

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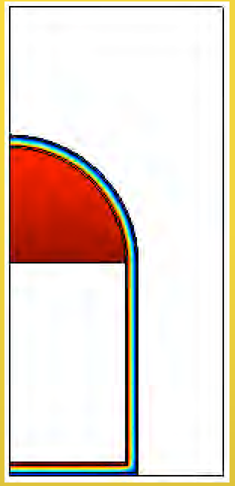

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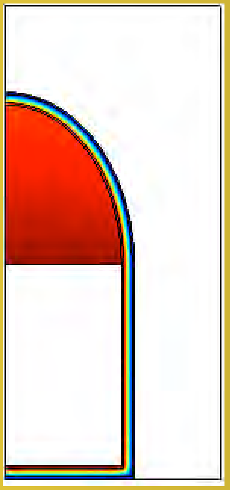

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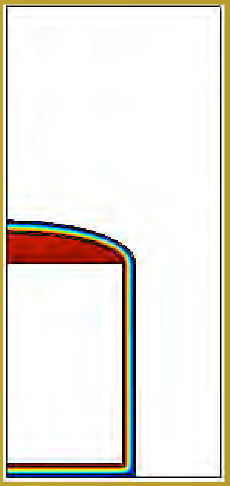

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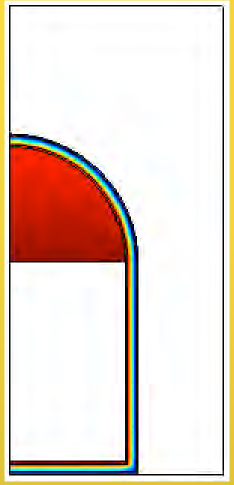

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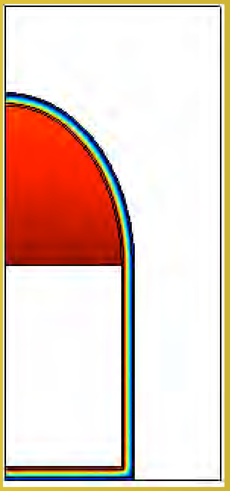

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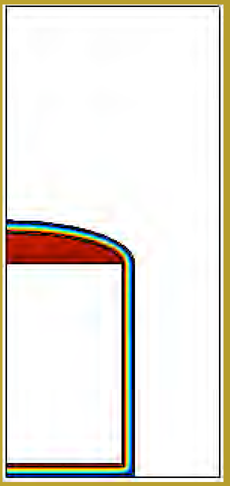

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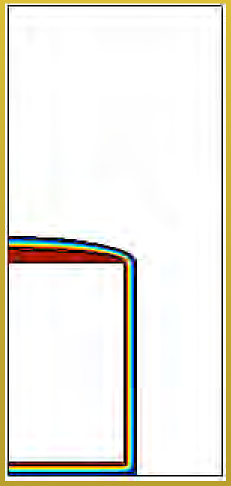

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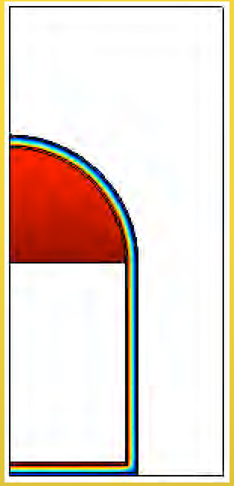

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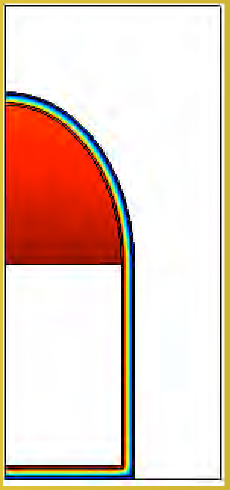

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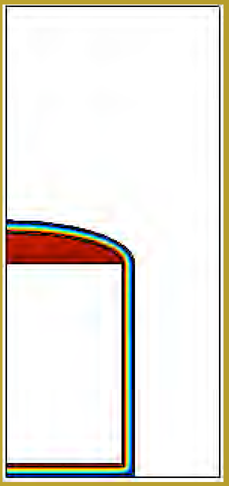
0.15[m]

C = Yes



Model 1

B/A = 0.20

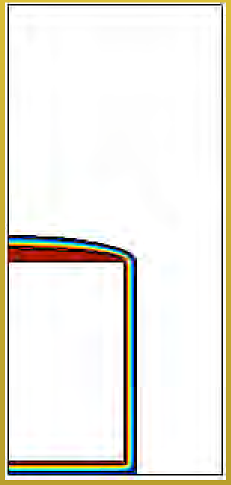
Length =

0.28[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 0.267

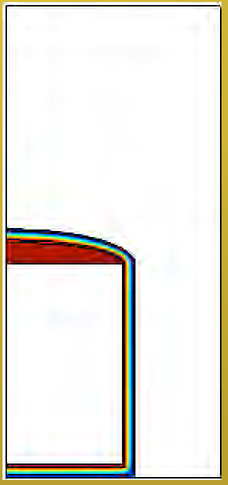
Length =

0.29[m]

Length2 =

0.15[m]

C = Yes



Model 1(HT):Solutions

Model 1

B/A = 1

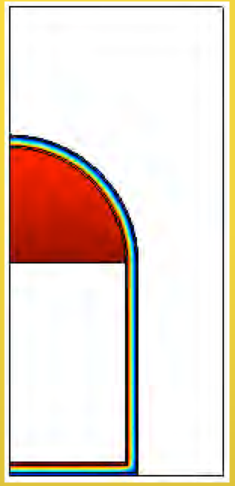
Length =

0.4[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 1.33

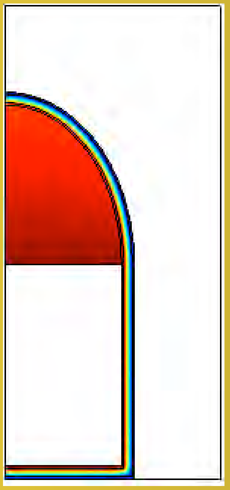
Length =

0.45[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 0.33

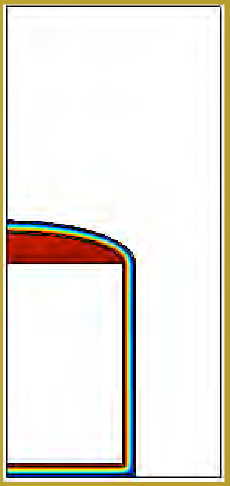
Length =

0.30[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 0.20

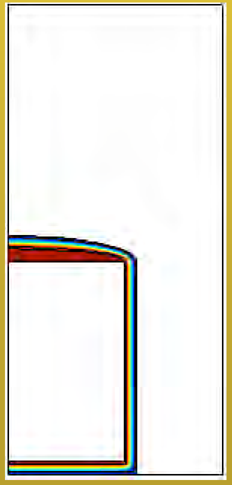
Length =

0.28[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 0.267

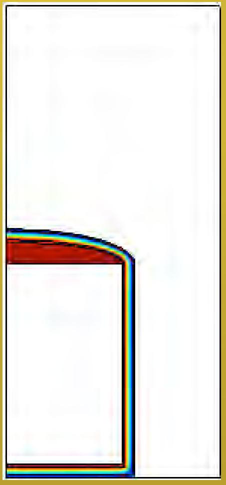
Length =

0.29[m]

Length2 =

0.15[m]

C = Yes



Model 1

B/A = 0.22

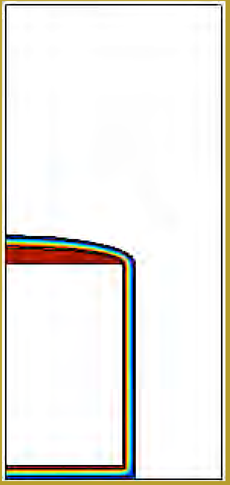
Length =

0.283[m]

Length2 =

0.15[m]

C = Yes



Model 2: Building a Navier-Stokes Multiphysics Model(s)

Do the following steps for each of the HT configurations (1-6):

Multiphysics>

Model Navigator>

Heat Transfer Module>

Weakly Compressible Navier-Stokes>

Steady-state analysis>

Click Add>

Click OK.

Setting up the Physics Subdomain in each Model 2 (N-S) configuration:

Subdomain #	Source	Material
1-5	Inactive	
6	BML	Air* F_z=F_buoyancy

>Click the Density tab>

*replace rho(p[... with rho((p+p_atm)[...

**(BML) Basic Materials Library

Model 2: Physics Settings HT Subdomain Settings

Do the following steps for each of the N-S configurations (1-6):

Multiphysics>General Heat Transfer>Physics>Subdomain settings
Select: Subdomain **6>**Select the **Active in this domain** check box.
Select: Air from the **Library material** list.

Model 2: Physics Settings HT Subdomain Settings

Do the following steps for each of the N-S configurations (1-6):

Multiphysics>General Heat Transfer>Physics>Subdomain settings
Select: Subdomain **6>**Select the **Active in this domain** check box.
Select: Air from the **Library material** list.

Click the **Convection** tab.
Select: Enable convective heat transfer check box.
Replace: rho(p[... with **rho((p+p_atm)[...**
In the **Density** edit window. Click **Apply**
Enter: u and **v** in the **Velocity field** edit windows.

Model 2: Physics Settings HT Subdomain Settings

Do the following steps for each of the N-S configurations (1-6):

Multiphysics>General Heat Transfer>Physics>Subdomain settings
Select: Subdomain **6>**Select the **Active in this domain** check box.
Select: Air from the **Library material** list.

Click the **Convection** tab.
Select: Enable convective heat transfer check box.
Replace: rho(p[... with **rho((p+p_atm)[...**
In the **Density** edit window. Click **Apply**
Enter: u and **v** in the **Velocity field** edit windows.

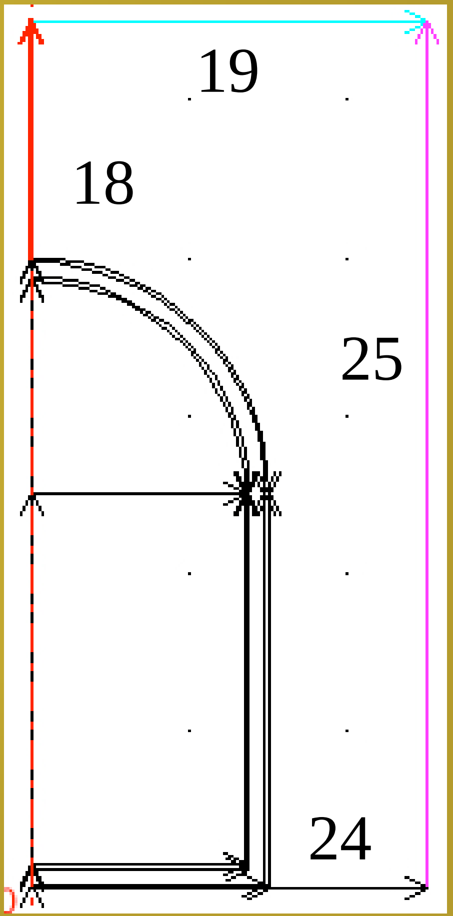
Click the **Artificial Diffusion** button.
Select: Streamline diffusion check box.
Select: Galerkin least-squares (GLS) from the list.

Model 2: Physics Settings
HT Boundary Conditions

Do the following steps for each of the N-S configurations (1-6):

Select: Multiphysics> General Heat Transfer> Physics>Boundary Settings.

Boundary	Condition
18	**Axial Symmetry**
19	**Convective Flux**
24	**Thermal Insulation**
25	**Temperature** Enter **T_amb** in the **T_0** edit window



Model 2: Physics Settings
N-S Boundary Conditions

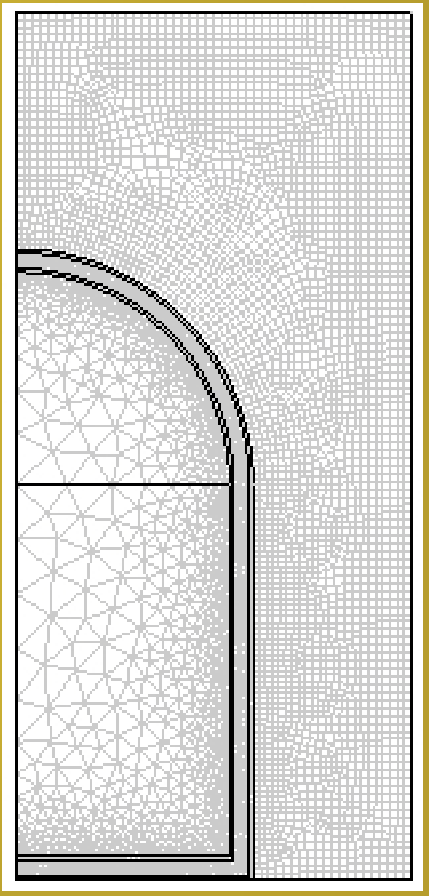
Do the following steps for each of the N-S configurations (1-6):

Select: Multiphysics> Weakly Compressible Navier-Stokes>Boundary Settings.

Boundary	Boundary Type	Boundary Condition
18	Symmetry boundary	Axial Symmetry
19	Outlet	Normal stress f_0 to **0**
25	Open boundary	Normal stress f_0 to **0**
23, 24, 29	Wall	No slip

Model 2 Mesh Generation:

Select: Mesh>Free Mesh Parameters
Select: Subdomain tab
In the **Subdomain selection** window, **Select:** Subdomain 6 >
Enter **0.005** in the **Subdomain mesh parameters** edit window>
Select: Quad>
Click the **Remesh** button>
Click OK



Quad Mesh generated for an Elliptically Shaped Tank Top B/A = 1.0

Solver Parameters:

Click the **Stationary** tab

Select: Damped Newton, Highly nonlinear problem, Manual tuning of damping parameters Check boxes

Solver Parameters:

Click the **Stationary** tab

Select: Damped Newton, Highly nonlinear problem, Manual tuning of damping parameters Check boxes

Enter:

Relative tolerance	**1.0E-2**
Maximum iterations	**50**
Initial damping factor	**1.0E-4**
Minimum damping factor	**1.0E-12**

Solver Parameters:

Click the **Stationary** tab
Select: Damped Newton, Highly nonlinear problem, Manual tuning of damping parameters Check boxes

Enter:

Relative tolerance	**1.0E-2**
Maximum iterations	**50**
Initial damping factor	**1.0E-4**
Minimum damping factor	**1.0E-12**

Click Advanced tab
Type of scaling **none**
Click OK
Select: Solve

Navier-Stokes Free Convection Solutions:

Models 2

B/A = 1

Length =

0.4[m]

Length2 =

0.15[m]

C = Yes

2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Navier-Stokes Free Convection Solutions:

Models 2

B/A = 1

Length =

0.4[m]

Length2 =

0.15[m]

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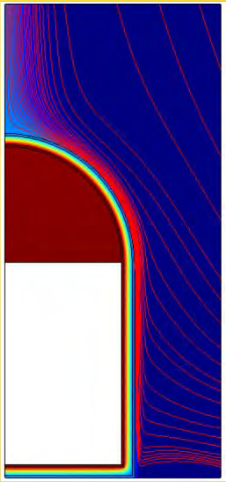


Models 2

B/A = 1.33

Length =

0.45[m]

Length2 =

0.15[m]

C = Yes

2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Navier-Stokes Free Convection Solutions:

Models 2

B/A = 1

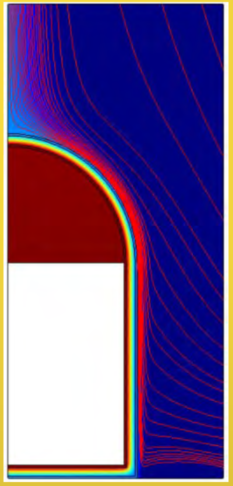
Length =

0.4[m]

Length2 =

0.15[m]

C = Yes



Models 2

B/A = 1.33

Length =

0.45[m]

Length2 =

0.15[m]

C = Yes

Model 2

B/A = 0.33

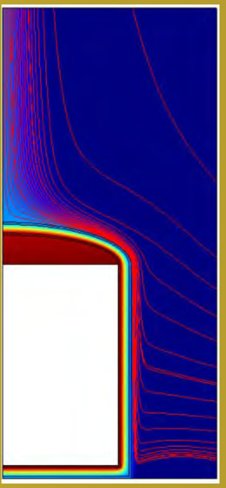
Length =

0.30[m]

Length2 =

0.15[m]

C = Yes



2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Navier-Stokes Free Convection Solutions:

Models 2

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Length =

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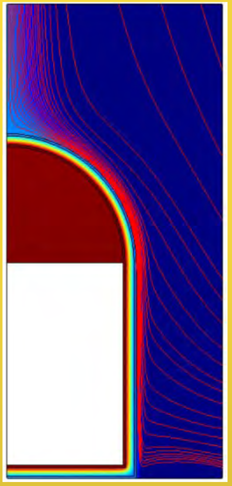

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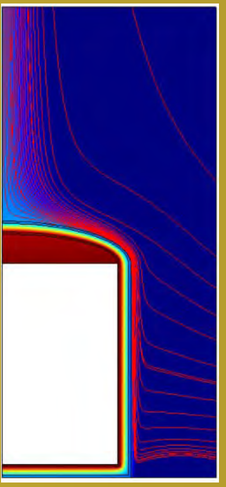


Model 2

B/A = 0.20

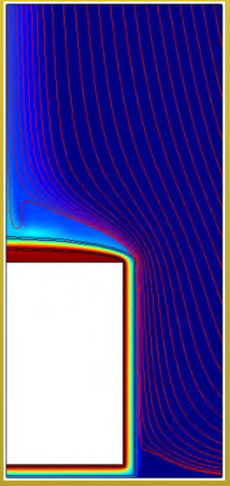
Length =

0.28[m]

Length2 =

0.15[m]

C = No



2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Navier-Stokes Free Convection Solutions:

Models 2

B/A = 1

Length =

0.4[m]

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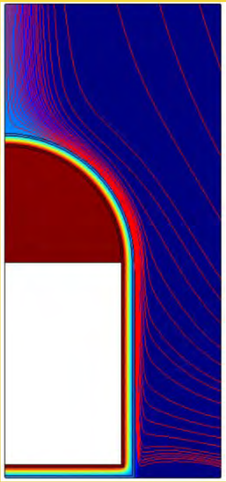
0.15[m]

C = Yes



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C = Yes

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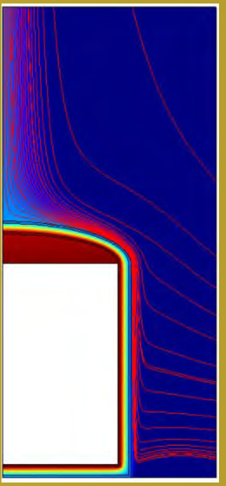

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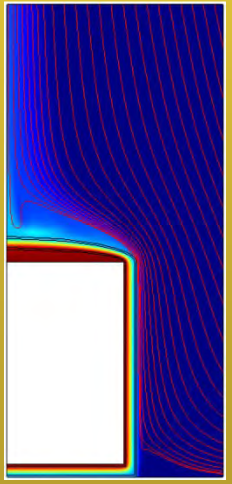
Length =

0.28[m]

Length2 =

0.15[m]

C = No



Model 2

B/A = 0.267

Length =

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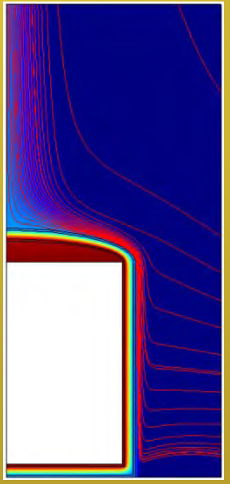
Length2 =

0.15[m]

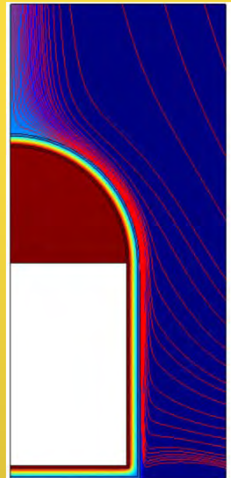
C = Yes



2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Navier-Stokes Free Convection Solutions:

Models 2
B/A = 1
Length =
0.4[m]
Length2 =
0.15[m]
C = Yes



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Length =
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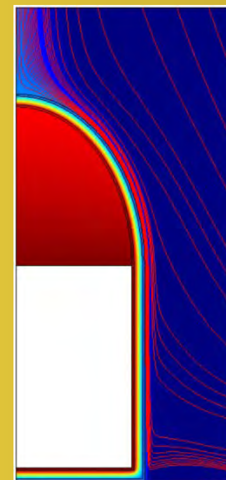

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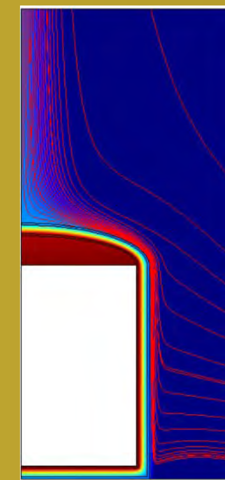
B/A = 0.33
Length =
0.30[m]
Length2 =
0.15[m]
C = Yes



Model 2
B/A = 0.20
Length =
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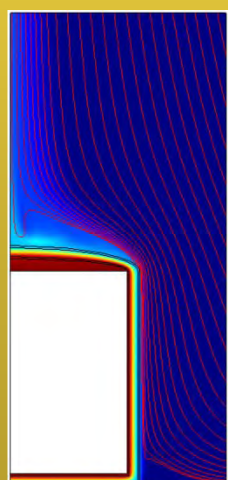
C = No



Model 2
B/A = 0.267
Length =
0.29[m]
Length2 =
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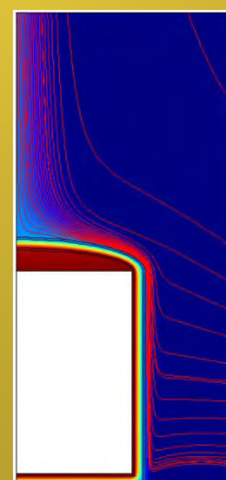
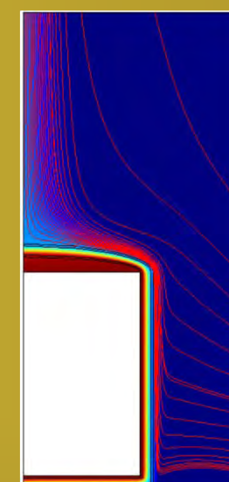
C = Yes



Model 2
B/A = 0.22
Length =
0.283[m]
Length2 =
0.15[m]
C = Yes

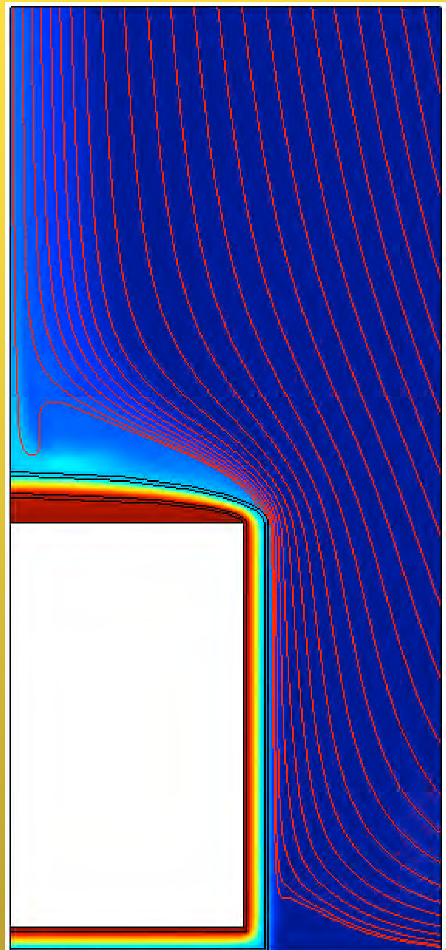


2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Roger W. Pryor, Ph.D.

Pryor Knowledge Systems

Conclusions



Model 2

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Length2 =

0.15[m]

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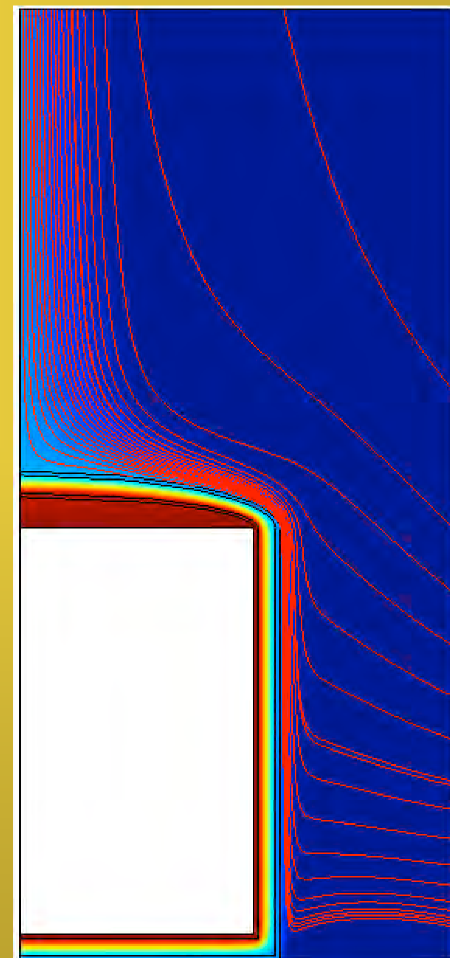

Successful implementation of a model is dependent on small parametric differences.

Non-Convergent

Convergent

2D Axisymmetric Elliptical Tank Top Model Cross-Sections

Thank you!