Finite Element Simulation of Microwave Structures for Experiments in Quantum Computing

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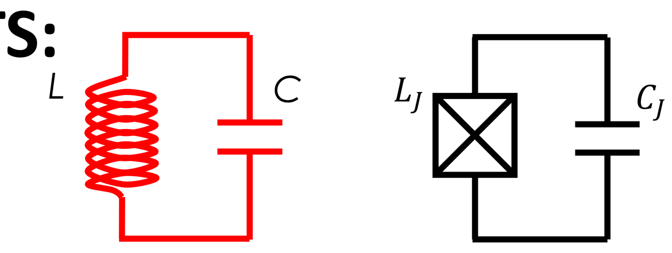
SUPERCONDUCITNG QUANTUM BITS:

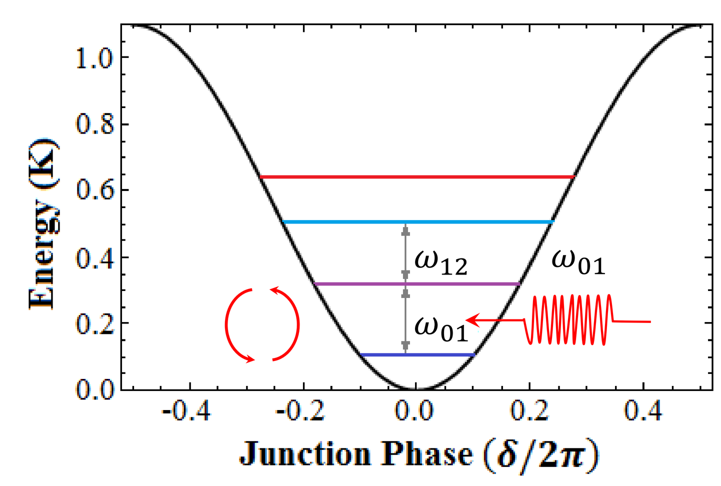
Harmonic Oscillator:

- Parabolic potential → Energy levels equally spaced
- Excitation → Coherent state

Anharmonic Oscillator:

- Cosine potential → Energy levels unequally spaced
- Excitation will coherently oscillate between bottom two level to form a **Quantum bit.**





Energy Vs Junction phase

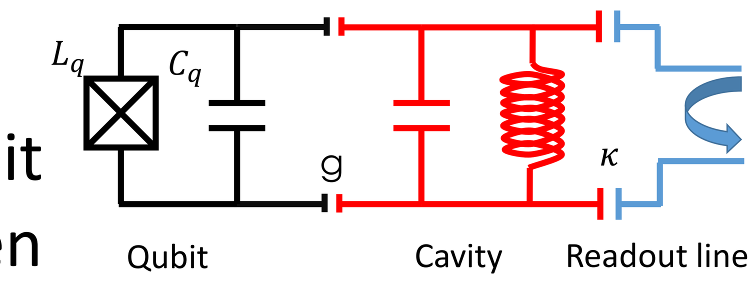
CIRCUIT QED:

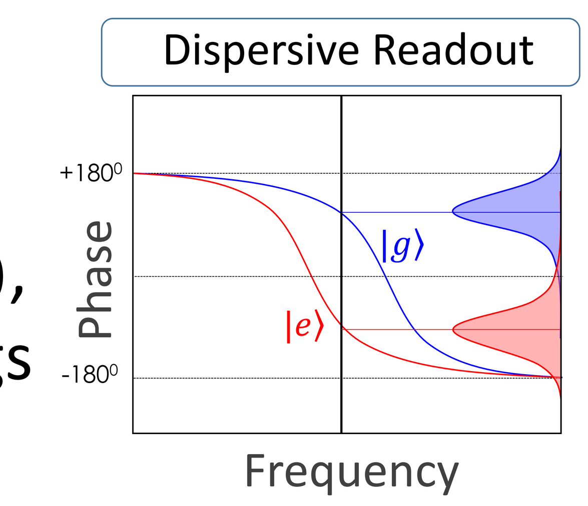
- The interaction between qubit and harmonic oscillator is given by Hamiltonian:

$$H_{JC} = \frac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar\omega_{cav}\hat{a}^\dagger\hat{a} + g\hat{\sigma}_x\hat{\sigma}_x$$

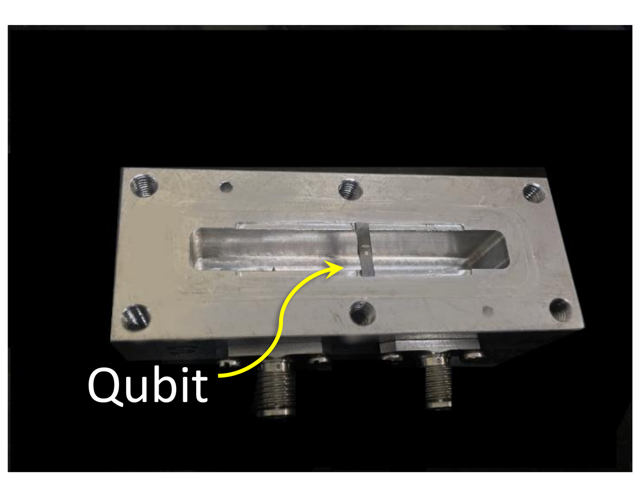
- In dispersive regime ($|\omega_q - \omega_{cav}| \gg g$), interaction is given by Jaynes-Cummings Hamiltonian:

$$H_{JC} = \frac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar(\omega_{cav} - \chi\hat{\sigma}_z)\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

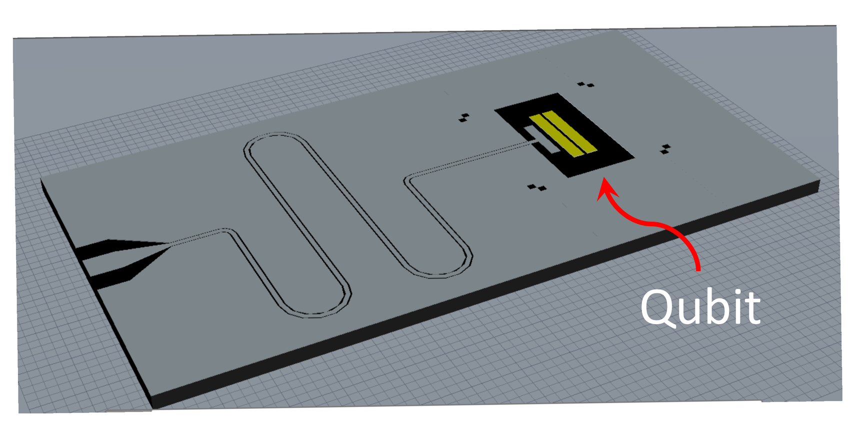




IMPLEMENTATION IN SUPERCONDUCTING QUBITS:



Qubit placed inside 3D waveguide cavity



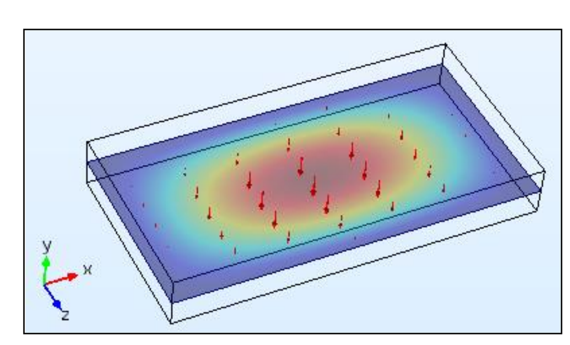
Qubit in 2D Transmission line cavity

- Qubit and cavity should lie in the 4-8 GHz band.
- Qubit-Cavity coupling and detuning should be adjusted for the validity of dispersive approximation.

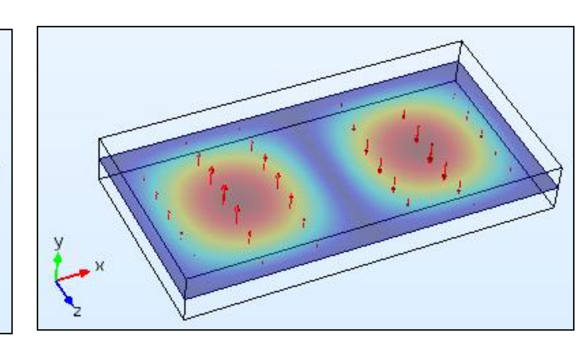
Qubit Frequency	Cavity Frequency	Qubit-Cavity coupling
~4-5 GHz	~7.0 – 7.5 GHz	80-100 MHz

Typical parameters for the Qubit+Cavity system

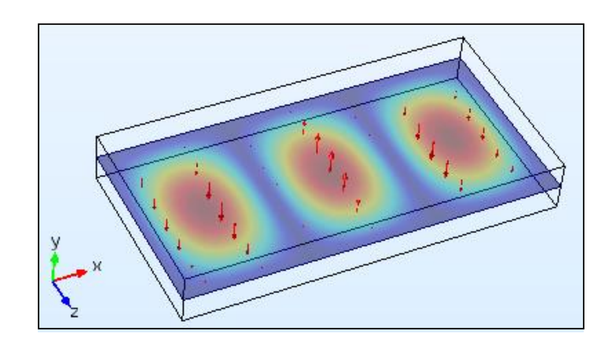
ELECTROMAGNETIC SIMULATION:



Fundamental mode of Cavity(7.3 GHZ)



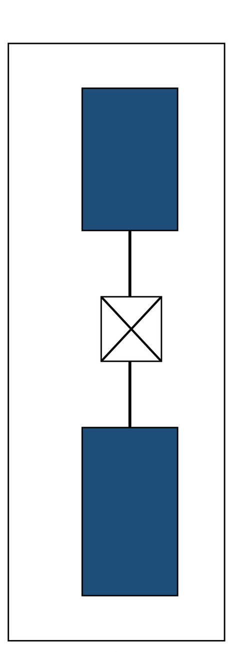
2nd harmonic mode of Cavity(9.3 GHz)



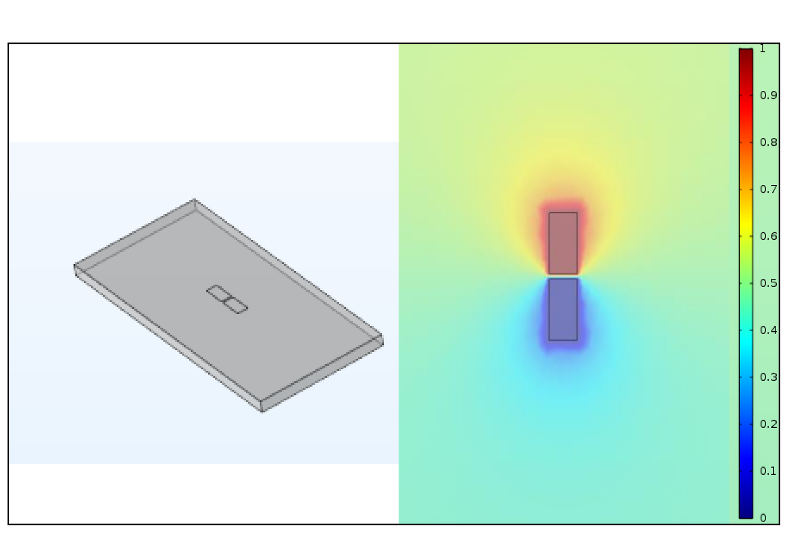
3rd harmonic mode of Cavity(11.93 GHz)

- Perfect electric boundary condition is used at the waveguide surface.
- Fundamental mode frequency is 7.3 GHz and other modes are well above such that one can ignore them by limiting the band of operation.

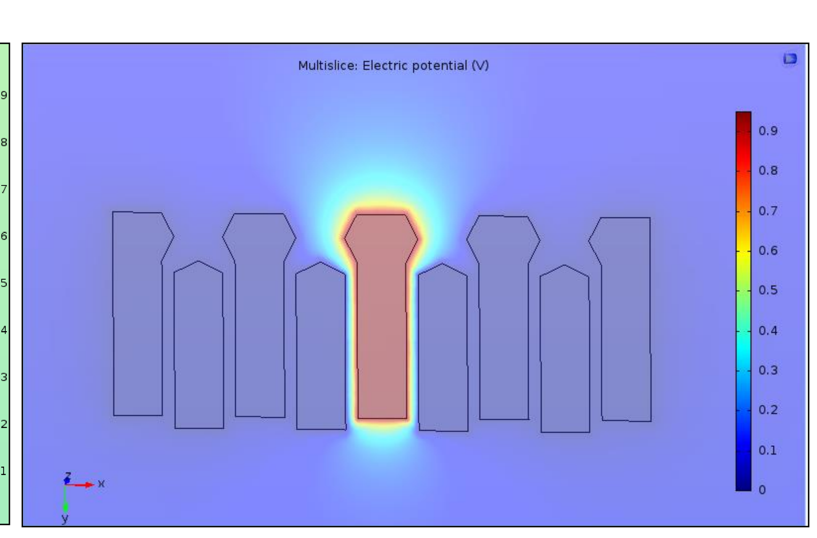
ELECTROSTATIC SIMULATION:



Qubit prototype



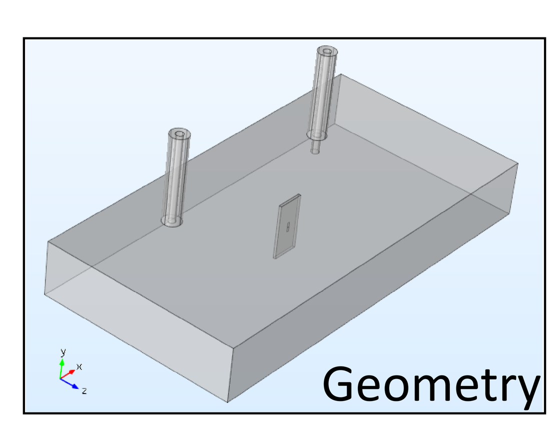
Qubit Capacitor pad design and electric potential distribution



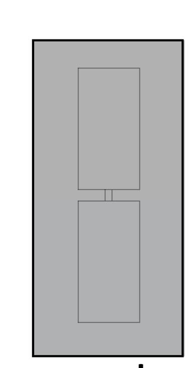
Multi-pad design and electric potential distribution

- Capacitance is estimated using Q = CV.
- Voltage terminals are defined at each qubit pad.
- C_{12} is estimated by computing charge Q_1 at terminal 1 by keeping voltage at terminal 1, $V_1 = 0$.
- This can be extended to the multi-pad structure and capacitances can be estimated using $Q_i = C_{ij} V_j$. All C_{ik}'s are estimated by keeping $V_k=1$ and rest at zero.

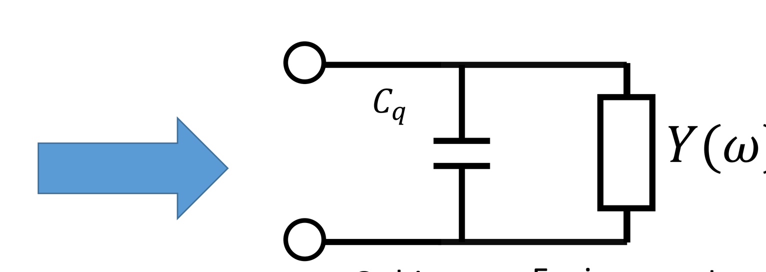
COUPLING ESTIMATION : PURCELL EFFECT



Geometry



Lumped port



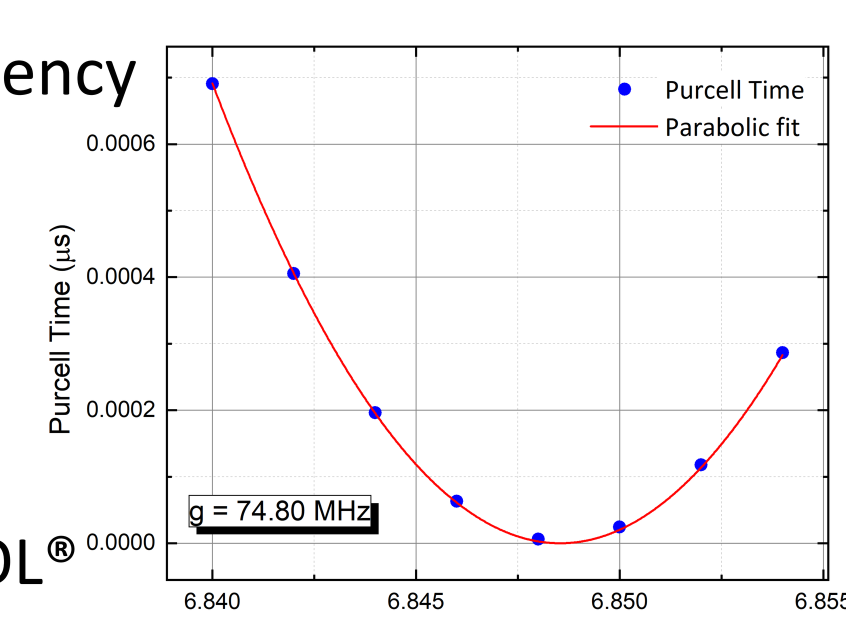
- Purcell time near cavity frequency has a parabolic form.

$$T_1^{Purcell} = \frac{1}{\kappa}\left(\frac{\omega-\omega_q}{g}\right)^2$$

- Excitation port is defined between qubit pads
- $Y(\omega)$ is computed using COMSOL® by terminating other ports with 50 Ω load.

$$T_1^{Purcell} = \frac{C_q}{Re[Y(\omega)]}$$

- Coupling is estimated by using parabolic fit for values extracted from COMSOL®.



Parabolic fit for values exported from COMSOL simulation

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[1]A. Wallraff, et.al. *Nature* **volume 431**, pages162–167 (2004)
[2]EM Purcell, Phys. Rev. **69**, 674 – Published 1 June 1946
[3] A.A. Houck, et.al. Phys. Rev.Lett. 101, 080502 (2008)