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Simulation of Deep Geothermal Heat Production

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Abstract: Geothermal heat production from deep reservoirs (5000-7000 m) is currently examined within the collaborative research program "Geothermal Energy and High-Performance Drilling" (gebo), funded by the Ministry of Science and Culture of Lower Saxony (Germany) and Baker Hughes. The projects concern exploration and characterization of geothermal reservoirs as well as production. They are gathered in the four major topic fields: geosystem, drilling, materials, and technical system. We present modelling of a reference set-up concerning the geothermal production itself.

Keywords: geothermics, pump and inject, Darcy's law, doublet

1. Introduction

Geothermal energy is world wide increasingly used as a CO_2-neutral, sustainable and renewable source of energy (see for example: Huenges *et al.* 2010). Geothermal facilities are increasingly utilized for heat and for power supply. A doublet, consisting of a pumping and an infiltration well, is a classical set-up, by which water of increased temperature is extracted from a near-surface or deep subsurface formation, and after heat utilization the cooled fluid is returned into the same aquifer horizon. The sketch in Figure 1 depicts several doublet designs. Red color represents hot water, i.e. production; blue color represents cold water, i.e. injection.

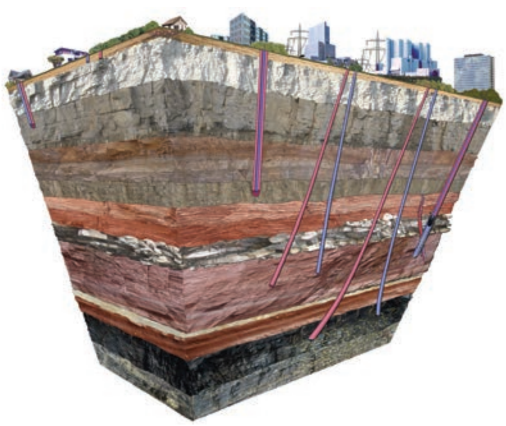


Figure 1. Doublet systems; source: Leibniz Institute for Applied Geophysics (LIAG)

2. Principle

In subsurface hydraulics a doublet system, consisting of a production point and an injection point is a well-established set-up for several different engineering purposes. In geothermal applications, fluid is extracted from a near-surface or deep subsurface formation in order to use its high heat content. The cooled fluid is usually re-injected into the same geological formation, which leads to a temperature decrease in the aquifer. Figure 2 depicts several single and double borehole designs, with several injection and pumping points.

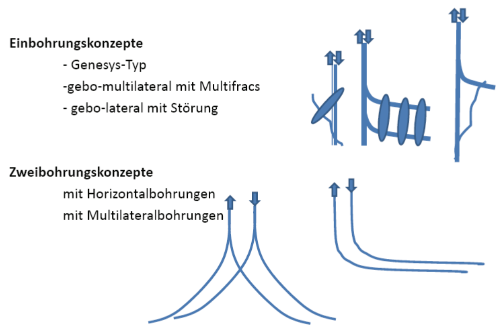


Figure 2. Different designs for deep geothermal heat production (Teodoriu, 2010)

A detail-view for a single doublet in a fracture is shown in Figure 3. It depicts a generic situation that is relevant for all different concepts. The complexity of the various concepts is reduced here to a single injection and a single production point.

The reference set-up consists of two pipes within a single borehole: one for pumping, i.e. for production of hot fluid and one for recharge, i.e. for injection of cold fluid. Figure 3 shows a sketch of a design, where a single borehole splits into two legs at a certain depth. The two legs are connected by highly permeable geological, natural or artificial strata in the deep subsurface. The colored part of the figure represents a permeable fracture that is surrounded by almost impermeable bed-rock (not shown). Broken lines indicate streamlines.

In the simulation approach we combine several models of different spatial dimension and complexity. We introduce four sub-domains:

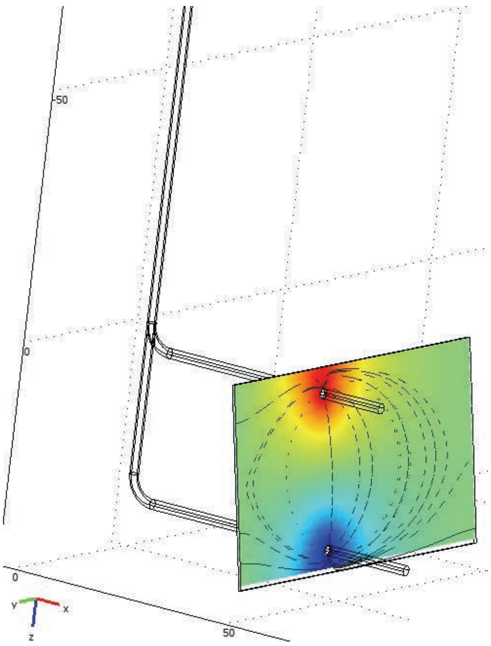


Figure 3. Conceptual sketch for geothermal heat production model

- the injection borehole
- the fracture
- the matrix
- the production borehole

The boreholes are represented by 1D model domains, the fracture is modelled in 2D and the surrounding bedrock is tackled in 3D. The model concerns fluid flow and heat transport in all sub-models; but in the bedrock only thermal processes have to be taken into account. We use either Dirichlet conditions (fixed pressure or temperature) or Neumann conditions (fixed fluid or heat flux) for the coupling between the sub-models.

There are some assumptions that simplify the model approach. Transversal heat flux in the pipes is neglected. The fracture is assumed to be porous and has a constant thickness. Moreover we consider the subsurface to be homogeneous, which really is seldom true in real systems. However, for a first approach these assumptions are surely justified and the major processes that determine the thermal system are considered.

3. Governing Equations

The mathematical algorithms used in the computations are implemented using analytical solutions based on potential theory. Darcy's law is given in Equation 1

$$\mathbf{v} = -K\nabla h \tag{1}$$

and is a well established empirical relationship for porous media flow (Bear 1976). It states the proportionality between filtration (or Darcy-) velocity **v** and the gradient of dynamic pressure. The latter is here represented by hydraulic head *h*, which is allowed if density changes can be neglected. The proportionality constant is the material parameter *K*, or the so-called hydraulic conductivity.

The principle of mass conservation for a 2D layer of thickness *H* is expressed in Equation 2:

$$\nabla \bullet H\mathbf{v} = 0 \tag{2}$$

(Strack 1989, Holzbecher 2011). Equations 1 and 2 can be combined to yield Equation 3:

$$\nabla \bullet HK\nabla h = 0 \tag{3}$$

Aquifer modeling in 2D is usually based on these differential equations (Holzbecher 2002). The differential equation (3) is formulated in terms of hydraulic head. An alternative formulation based on the hydraulic potential φ is given in Equation 4:

$$\varphi = K \cdot H \cdot h - \tfrac{1}{2} K \cdot H^2 + \varphi_0 \tag{4}$$

for which the classical *potential equation* or *Laplace equation* holds (Strack 1989, Holzbecher 2011).

$$\nabla^2 \varphi = 0 \tag{5}$$

The potential φ has the physical unit of [m^3/s]. Usually, the constant φ_0 has to be chosen appropriately in order to fulfil a point condition for hydraulic head. We utilize it here for flow in the fracture.

In the implementation of the program we use the complex potential, which is given by Equation 6:

$$\Phi = \varphi + i\psi \tag{6}$$

with imaginary unit *i* and streamfunction ψ. This is an extended form of the real potential φ. The streamfunction has several interesting features. Concerning the visualization of flow patterns, the advantage of the streamfunction is that streamlines are identical to contour lines of ψ. Streamline patterns are visualized as contour lines of the streamfunction.

In the implementation, we also utilize that the complex potential is represented as a function defined on the complex plane by $z=x+iy$. We identify the model region with a part of the complex plane.

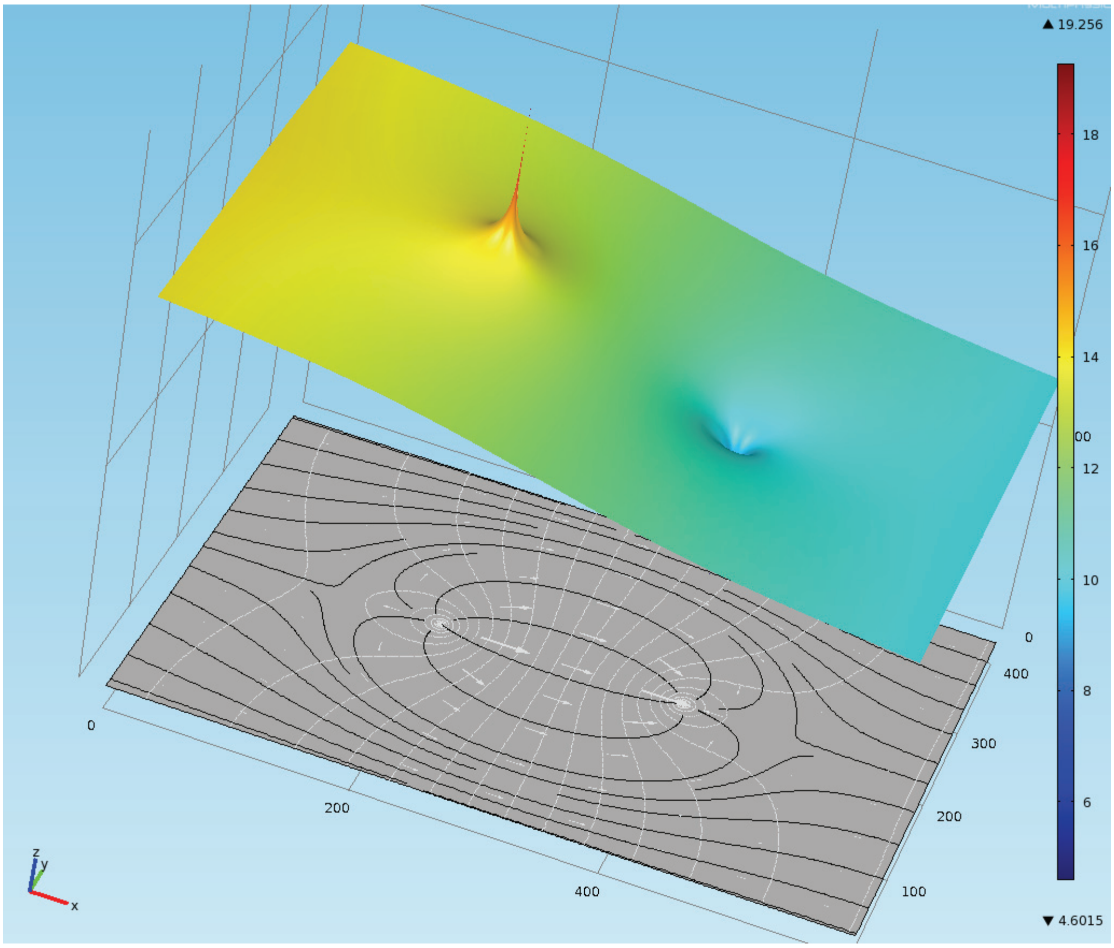


Figure 4. Doublet: piezometric head as surface plot, streamlines and isopotential lines

Two types of solutions of the potential Φ are implemented here. The analytic solution for a regional 1D flow field is given in Equation 7:

$$\Phi = \bar{Q}_0 z \tag{7}$$

where Q_0 is the baseflow discharge vector given by a complex number. Real and imaginary parts represent the vector components of baseflow in the x and y directions, which have units of m^3/s. The overbar denotes a complex conjugate.

The solution for a well with a pumping rate Q_{well} at the position z_{well} is given in Equation 8:

$$\Phi = \frac{Q_{well}}{2\pi} \log\left(z - z_{well}\right) \tag{8}$$

According to the principle of superposition, solutions for generic situations can be summed up as analytical elements to obtain a solution for a specific situation. Here we take the baseflow element (7) and add a function of type (8) for each well. A single doublet in a constant flow field is thus given in Equation 9:

$$\Phi = \bar{Q}_0 z + \frac{Q_{well}}{2\pi}\left[\log\left(z - z_{well1}\right) - \log\left(z - z_{well2}\right)\right] \tag{9}$$

Figure 4 depicts doublet flow for a single doublet, calculated by COMSOL Multiphysics.

In the model we solve the differential equation for heat transport

$$\frac{\partial T}{\partial t} = \nabla D \nabla T - \kappa \mathbf{v} \nabla T \tag{10}$$

for temperature T as dependent variable in various subdomains. In the tube, **v** corresponds with the mean flow velocity. In the fracture **v** is the Darcy velocity and is calculated according to equation (1). In the matrix we assume negligible advective flux and set velocity to zero, i.e. let last term in equation (11) vanish. D represents the thermal diffusivities, which usually depends on the subdomain. κ denotes the ratio of heat capacities:

$$\kappa = \rho_f C_f / \rho_{sf} C_{sf} \tag{11}$$

with densities ρ and heat capacities C. Subscripts f denote fluid, sf the solid/fluid system.

4. Use of COMSOL Multiphysics

For modelling we used COMSOL Multiphysics 3.5 up to 4.2 to set up the model. For the 2D fracture model we utilize the analytical solution, obtained from Darcy's law (1), to compute the velocity field.

The temperature field in the four subdomains is calculated using either:

- heat transfer mode (*ht*)
- or: general pde-model (*g*)

In the general pde-mode we use the equation

$$\frac{\partial T}{\partial t} = -\nabla \bullet \mathbf{\Gamma} + j \tag{12}$$

with heat source/sink term j and flux vector

$$\mathbf{\Gamma} = -D\nabla T + \kappa \mathbf{v} T \tag{13}$$

4.1 Geometry and Meshing

We use triangular meshes in 2D, tetrahedra meshes in 3D. The 2D model is refined near both the injection and production positions. The 3D mesh is refined near the fracture. For some model runs we use adaptive meshing in the fracture domain. We used physics-controlled meshing, as the linear transformations between the 2D and the 3D geometry may lead to false interpretations if the user-controlled meshing is applied.

Parameters concerning the geometry of the system are given in Table 1.

4.2 Boundary and Initial Conditions

For the injection tube we use a Dirichlet condition at the inlet. The bottom of the tube corresponds with the connection to the fracture. In the 1D model we require the typical outflow Neumann type boundary condition:

$$\frac{\partial T}{\partial n} = 0 \tag{14}$$

where n denotes the normal direction.

For the 2D model of the fracture we use a Dirichlet condition at the injection position. The value is taken as a result from the 1D model for the injection tube, i.e. the computed temperature at the outlet. At the position of the production well we use also the outflow condition (14).

For the production tube we use a Dirichlet condition at the bottom. The value is taken from the 2D fracture model, in which it is calculated. At the top we again use the Neumann condition (14). The construction of the entire model thus delivers a breakthrough curve for temperatures at the outlet of the production tube.

As described the model has links from the injection tube to the fracture, and from the fracture to the production tube. In addition there is a 2-way coupling between the fracture and the surrounding matrix.

The temperature values within the fractures are used as Dirichlet boundary conditions for the temperature calculations within the matrix. At all other boundaries we require no-flow condition (12). Internally a heat flux

$$\mathbf{j} = -2D\frac{\partial T}{\partial n} \tag{15}$$

is calculated within the 3D matrix domain and is taken as a flux term in the 2D fracture model. Thus fracture and matrix model have to be calculated in fully coupled mode. As the heat flux from the matrix reaches the fracture from two directions, we have to use the factor 2 in equation (15).

As initial condition we require a high temperature in the fracture and the matrix. For the tubes we use the geothermal gradient as an initial condition.

4.3 Input parameters

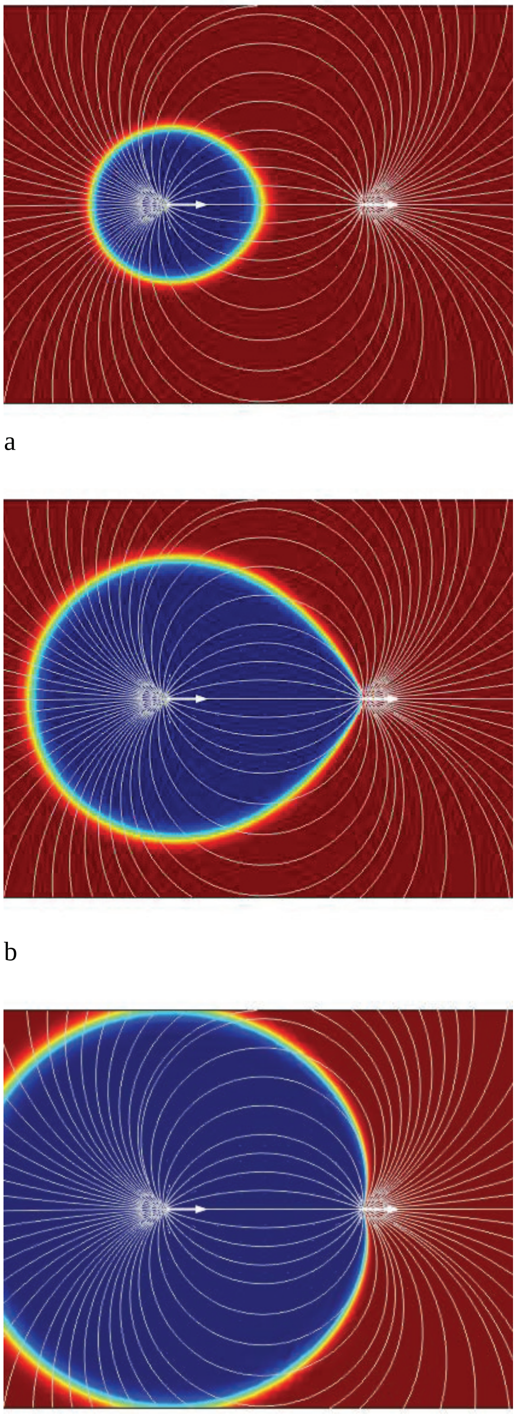
The input parameters for the reference model are gathered in Table 1. These concern physical as well as geometrical parameters. The reference case was set-up as a benchmark to co-operation with Technical University of Braunschweig and will be used for intercomparison of results with the cellular automata method.

Table 1: Input parameter values for model set-up

Parameters		
Name	***Value***	***Unit***
Inlet temperature	70	°C
Reservoir temperature	200	°C
Reservoir depth	5000	m
Geothermal gradient	0.038	°C/m
Doublet distance	100	m
Borehole diameters	9 5/8, 7	inch
Total pumping/ injection rate	0.115	m^3/s
Velocities in boreholes	0.62, 1.16	m/s
Fracture length	250	m
Fracture density	2000	kg/m^3
Fracture porosity	0.6	-
Fracture hydraulic conductivity	$1.5\ 10^{-2}$	m/s
Fracture heat capacity	1000	J/kg/ °C
Fracture thickness	0.002	m
Longitudinal dispersivity	1	m
Transversal dispersivity	0.1	m
Matrix thickness	5	m
Matrix thermal conductivity	2	W/m/° C
Time period	$1.5\ 10^6$	s

5. Results and Discussion

Figures 3a,b,c depict temperature distributions in the fracture at different stages of production. In the initial phase (Subfigure 3a) the cold plume covers the vicinity of the injection borehole only. The cold plume is very near to circular shape. In a further stage the front of the plume reaches the production borehole (Subfigure 3b). In that phase a mixture of cold and hot water is pumped in the production borehole. Depending on heat requirements the temperature level may not be sufficient to guarantee an economical management of the facility.



a

b

c

Figures 3. Streamlines (white) and temperature distributions (blue = cold, red = hot) in fracture at early (a), intermediate (b) and late (c) stage of production

If the system is operated further the cold plume is increasing further, as shown in Subfigure 3c. The share of cold water in production is also increasing, accompanied by lowering the temperature of the pumped liquid. At a certain time instant the system becomes uneconomical.

5. Conclusions

We presented a model set-up, by which a geothermal production system can be studied using a combined 1D/2D/3D approach. The connection between the different sub-models (boreholes, fracture, matrix) is given by linear transformations between sub-model variables and boundary conditions.

The method is superior to the often applied suggestion of Vinsome & Westerveld (1980), in which a simple formula for the prediction of heat flux from the rock matrix to permeable stratum is developed. However, the formula is derived under the assumption that heat fluxes parallel to the permeable stratum, here the fracture, can be neglected. Such an assumption is attributed to the fact that the computational effort to consider such 'transversal' fluxes is quite high. However the here proposed approach is not based on such an assumption.

However the coupling between 2D and 3D geometries as suggested above is not easy. The scale difference between fast advective processes in the permeable stratum and relatively slow diffusion processes between matrix and fracture remains a challenge for numerical modeling. We intend to examine this point in more details in further simulations.

The reference simulation, described herein, is used for comparison with models of other project groups within the gebo-project (see: gebo), which work with alternative numerical concepts. An intercomparison is intended.

The presented approach can be extended easily to more complex situations. Thus it may be relevant to consider baseflux in the permeable stratum, for example in case of high pressure gradients and corresponding high fluxes in a fracture. Some first own results were already published by Holzbecher & Sauter (2010) and more is in preparation (Holzbecher & Sauter 2011).

6. References

1. Bear, J., *Hydraulics of Groundwater*, Mc Graw Hill, New York (1976)
2. gebo, Geothermal Energy and High Performance Drilling Research Program gebo, see website: http://www.gebo-nds.de/en/
3. Holzbecher E., *Groundwater Modelling - Simulation of Groundwater Flow and Pollution*, Electronic Book on CD-ROM, FiatLux Publ., Fremont (USA)
4. Holzbecher, E., *Environmental Modeling*, Springer Publ., Heidelberg, 2nd ed. (2011)
5. Holzbecher E., Sauter M., The wells-simulator: analytical solutions for groundwater flow using modified streamfunction coutouring, *Geoenvironmental Engineering and Geotechnics*, ASCE Geotech. Spec. Publ. No. 204, 194-201, Geo-Shanghai (2010)
6. Holzbecher E., Sauter M., Geothermal Operation - Development and Simulation of Well Configuration Tools, Intermediate report project G7, internal paper (2011)
7. Huenges E. (ed.), *Geothermal Energy Systems – Exploration, Development and Utilization*, Weinheim, Wiley-CVH (2010)
8. Strack, ODL., *Groundwater Mechanics*., Prentice Hall, Englewood Cliffs (1989)
9. Teodoriu C., Bohrtechnik, Vortrag gebo-Jour Fixe, 13.8.2010
10. Vinsome P.K.W., Westerveld J., A simple method for predicting cap and base rock heat losses in thermal reservoir simulators, *J. Can. Petr. Tech.*, Vol. 19(3), 87-90 (1980)

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