Time Varying Nonlinear Schrödinger Equation: Bose-Einstine Condensation via Gross-Pitaevskii Eq

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Introduction: Find the quantum mechanics wave function Ψ(x,y,z,t) as a solution to the Nonlinear Schrödinger equation via the Gross-Pitaevskii equation. Ψ represents a typical boson particle (near zero K T) as it interacts with N like neighboring ones found in a dilute gas of ground state bosons.

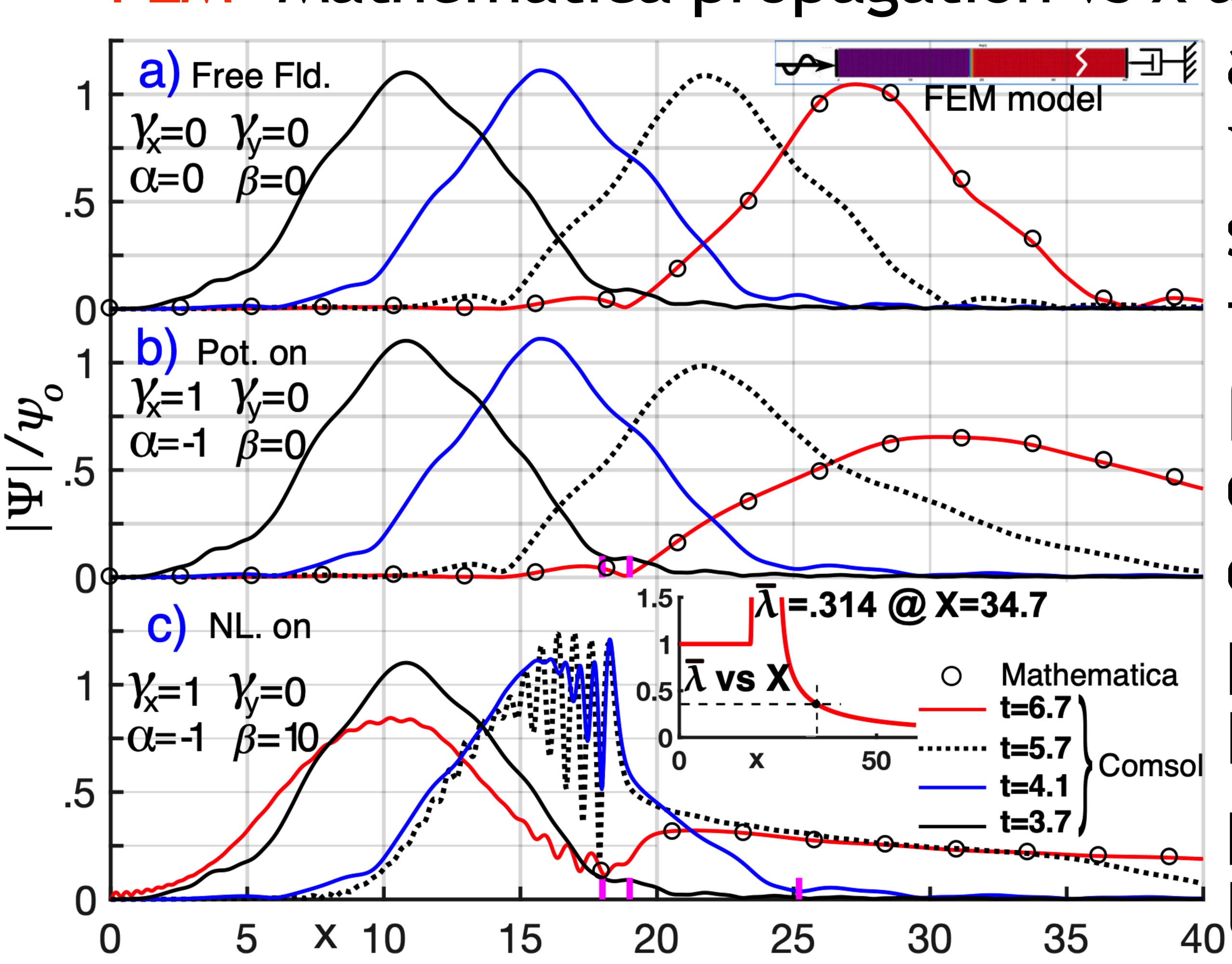
Computational Method: The 2D Nonlinear Schrödinger Eq.(1) for the behavior of a cold boson particle [1] in terms of non dimensional variables Ψ,x,y,t with V potential and nonlinear β multiplier control parameters are solved with COMSOL'S *"General-Form PDE"*.

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}\right)+\Psi\left\{V(x,y)+\beta f(\Psi)\right\} \quad (1)$$

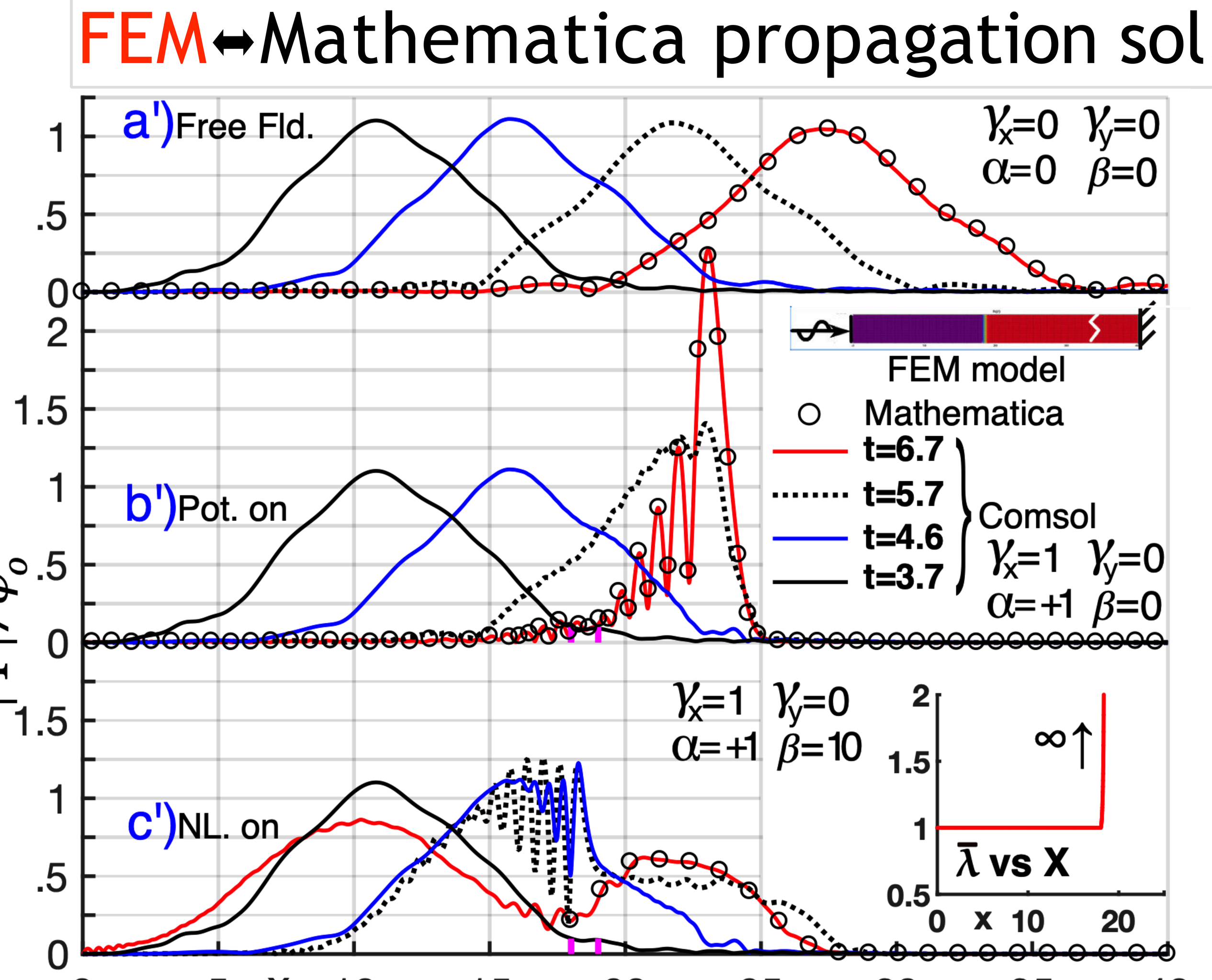
$$V(x,y) = \alpha\frac{1}{2}\left(\gamma_x^2(x-x_o)^2+\gamma_y^2(y-y_o)^2\right) \quad (2)$$

$$f(\Psi) = |\Psi|^2 \quad (3)$$

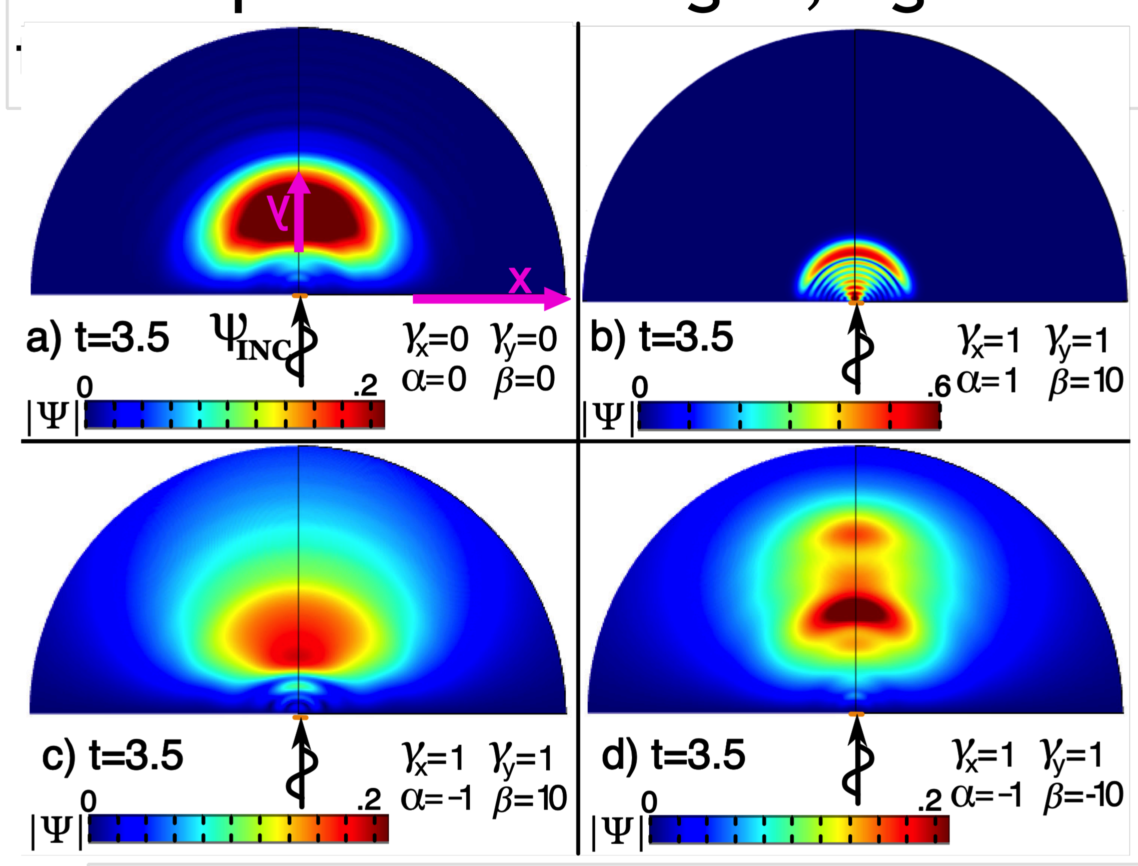
Results: • **Fig.1 <u>PW Pulse in V(x)<0 Field</u>** below validates the $\Psi=\Psi_o e^{-i\omega t}$ end driven *Wave Guide* COMSOL FEM↔Mathematica propagation vs x and is shown for $V<0$ and β > 0. The |Ψ|/Ψ_o vs x for 4 time snapshots is shown for: a′) V potential & β term off, b′)Turn on potential only, c′) Turn on both potential & β term. Local k-ω → allowable propagating $\bar{\lambda}$ wave lengths @ Fig.1c inset.



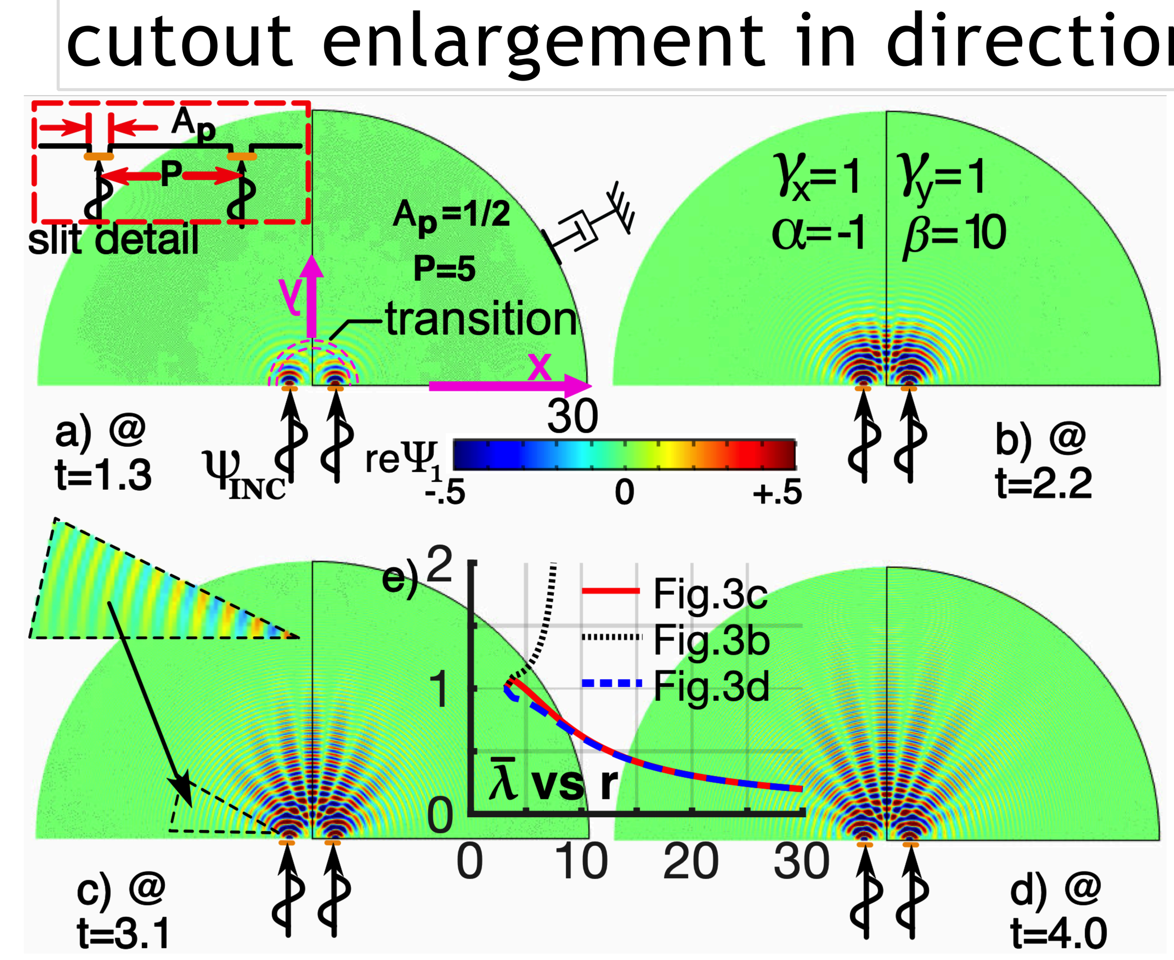
• **Fig.2 <u>PW Pulse in V(x)>0 Field</u>** below validates the $\Psi=\Psi_o e^{-i\omega t}$ end driven *Wave Guide* COMSOL FEM↔Mathematica propagation sol. vs x and is shown for V>0 & β>0. The |Ψ|/Ψ_o vs x for 4 time snapshots is shown for: a) Potential & β term off, b)Turn on potential only, c) Turn on both potential & β term. Local k-ω → allowable propagating $\bar{\lambda}$ wave lengths (see inset).



• **Fig.3 <u>PW Pulse thru one slit</u>** upper right is a 2D version of the previous 1D Bar models. A free field PW pulse (with zero V Potential and zero NL β term turned off) impinges on a baffle with 1 slit, and enters a small circular 2D free field zone surrounding the slit. Beyond this circular region, the V potential term and NL β term are gradually turned on with a ∫ shaped step function. Fig.3a is the 2D classic Free Field Schrödinger counterpart of the 1D Fig.1a; Fig.3c is the 2D counterpart of the 1D Fig.1c; Fig.3d is the same as Fig.3c except the sign of the NL β term is negative; Fig.3b is the 2D counterpart of the 1D Fig.2c′. For local cylindrical wave k-ω → allowable propagating $\bar{\lambda}$ wave lengths, see the Fig.4 inset.



• **Fig.4 <u>PW Pulse thru two slits</u>** below is the same as 1 slit Fig3c, except the PW passes through 2 slits. The idea here is to show how two nonlinear wave functions Ψ_1 & Ψ_2 interact with each other as they emerge from the slits. The aperture and pitch of the slits are shown in the Fig.4a inset. A radial absorbing BC is used at the outer circular model boundary. Bands of constructive & destructive interference are tracked in a four time snapshot sequence {1.3,2.2,3.1,4} where Figs.(4a-d) show a time growth of the reΨ_1 component. The red local wavelength $\bar{\lambda}$ vs r plot (Fig.4e inset), predicts traveling cylindrical waves, and at a decreasing wavelength (e.g. Fig.4c inset triangular cutout enlargement in direction of propagation illustrates the yellow banded peak to peak spans getting shorter in +r direction). Comparing the 1 slit Fig.3c and the 2 slit Fig.4c results, illustrates completely different field responses.



Conclusions: The *General-Form PDE* option solved the NL Schrödinger Eq. Agreement between COMSOL and an alternate FEM code for long 1-D models in a PW waveguide is obtained. The local k-ω dispersion relation gives an estimate of the expected spatial wavelengths at a given ω which is useful in selecting mesh sizes & applying absorbing BC's.

References:1. Xavierc A, Et. Al. ,"Comp. Methods for the Dynamics of the NLSE / GPE", Computer Physics Comm. 00 (2013)