Modeling Flow and Deformation During Hot Air Puffing of Single Rice Kernels

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Introduction

- When rice is subjected to intense heating (using hot air), it results in **rapid evaporation** of liquid water inside to form vapor
- As a consequence, **large pressures are generated** within the kernel in a span of 15s resulting in **large volume changes** of the kernel thereby causing it to puff rapidly
- Under suitable conditions, the ratio of initial volume to volume after puffing could be as high as 10; a higher expansion ratio indicates a better quality product
- Rice puffing process is a complex interplay of **mass, momentum and energy** transport along with **large volumetric expansion** of the solid matrix.
- By treating rice as a porous material, a **fundamentals-based model of rice puffing process** that can describe heat and moisture transport, rapid evaporation and large deformations of the solid matrix is presented to understand the factors affecting the puffing process

Modeling Framework

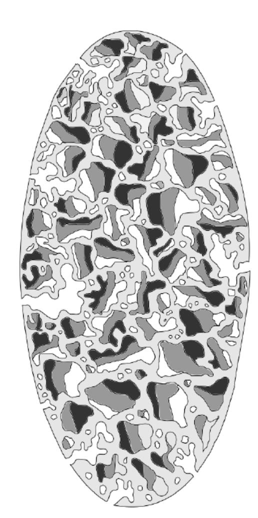
Transport Model

- Conservation of energy
  - Bulk flow
  - Conduction
  - Phase change
- Conservation of mass
  - **Water**: bulk flow, capillary flow and phase change
  - **Gas** (air, vapor): bulk flow phase change, binary diffusion
- Conservation of momentum
  - Darcy's flow

Solid Mechanics Model

- Linear momentum balance
  - Lagrangian description
  - Gas pressure gradients
- Constitutive law
  - Material non-linearity
  - **Mooney-Rivlin** hyperelastic material
  - **Perfectly Plastic** material
  - Von-Mises stress criteria for plastic deformation

Rice kernel treated as a porous, elasto-plastic solid



VARIABLES OF INTEREST

Gas Pressure inside the rice kernel, Amount of **Evaporation**, Degree of **Volumetric Expansion** and Extent of **Plastic Deformation**

Governing Equations: Transport Model

Conservation of Mass

Water:

$$\frac{\partial c_w}{\partial t} + \underbrace{\left(\mathbf{v_w} - \mathbf{v_s}\right)\cdot\nabla c_w + c_w \nabla\cdot\mathbf{v_w}}_{\text{Convection}} = \underbrace{\nabla\cdot\left(D_w \nabla c_w\right)}_{\text{Diffusion}} - \underbrace{\dot{I}}_{\text{Phase change}}$$

Gas:

$$\frac{\partial c_g}{\partial t} + \left(\mathbf{v_g} - \mathbf{v_s}\right)\cdot\nabla c_g + c_g \nabla\cdot\mathbf{v_g} = \dot{I}$$

Vapor:

$$\frac{\partial c_v}{\partial t} + \left(\mathbf{v_g} - \mathbf{v_s}\right)\cdot\nabla c_g + c_g \nabla\cdot\mathbf{v_g} = \nabla\cdot\underbrace{\left(\varphi S_g \frac{C^2}{\rho_g} M_a M_v D_{eff,g} \nabla x_v\right)}_{\text{Binary diffusion (vapor and air)}} + \dot{I}$$

Conservation of Energy

$$\frac{\partial}{\partial t}\left[\sum_{i=s,w,v,a}\left(c_i c_{p,i} T\right)\right] + \underbrace{\sum_{i=w,v,a}\left(\mathbf{v_i} - \mathbf{v_s}\right)\cdot\nabla\left(c_i c_{p,i} T\right) + \sum_{i=w,v,a}\left(c_i c_{p,i} T \nabla\cdot\mathbf{v_i}\right) - c_{p,w} T \nabla\cdot\left(D_c \nabla c_w\right)}_{\text{Convection}} = \nabla\underbrace{\left(k_{eff}\nabla T\right)}_{\text{Conduction}} - \lambda\dot{I}$$

Conservation of Momentum

Darcy's Law: (water and gas)

$$\mathbf{v}_i - \mathbf{v}_s = -\frac{k_i k_{r,i}}{S_i \phi_i \mu_i}\nabla P$$

Governing Equations: Solid Mechanics Model

Balance of Linear Momentum

Deformation due to gas pressure gradient :

$$\nabla\sigma = \nabla P \qquad \underbrace{\sigma'}_{\text{Cauchy Stress}} = J^{-1}\mathbf{F}_{\mathbf{el}}.\mathbf{S}.\mathbf{F}_{\mathbf{el}}^T, \qquad \underbrace{\mathbf{S}}_{\text{2nd PK Stress}} = \frac{\partial W_S}{\partial \mathbf{E}_{\mathbf{el}}}$$

Large Deformations:

$$\mathbf{E}_{\mathbf{el}} = \frac{1}{2}\left[(\nabla.\underbrace{\mathbf{u}}_{\text{Solid displacement}})^T + \nabla.\mathbf{u} + (\nabla.\mathbf{u})^T\nabla.\mathbf{u}\right]$$

Constitutive Law

Mooney-Rivlin Hyperelastic

$$W_s = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + \frac{1}{2}\kappa(J_{el} - 1)^2$$

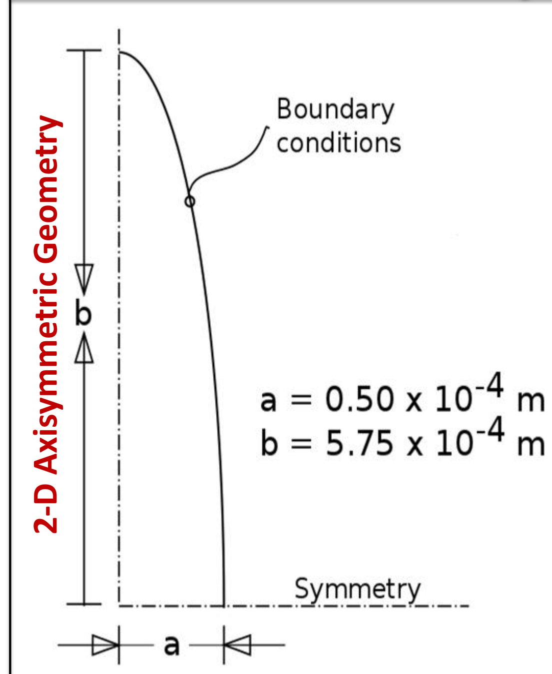
Von-mises Yield Crieteria:

$$F = \sigma_{mises} - \overbrace{\sigma_{YS}}^{\text{Yield Stress}}$$

Perfectly Plastic Material:

$$\sigma_{YS} = \sigma_{YS0}$$

Geometry & COMSOL Implementation



Boundary Conditions

- Forced convection heat transfer
- Moisture loss through evaporation
- Free surfaces for deformation

Symmetry

- No axial displacement
- Insulated for Energy and Moisture Transport

COMSOL Implementation

- Transport of Dilute Species
- Transport of Concentrated Species
- Heat Transfer in Fluids
- Darcy Law
- Solid Mechanics

Primary Variables

Heat and Mass Transport

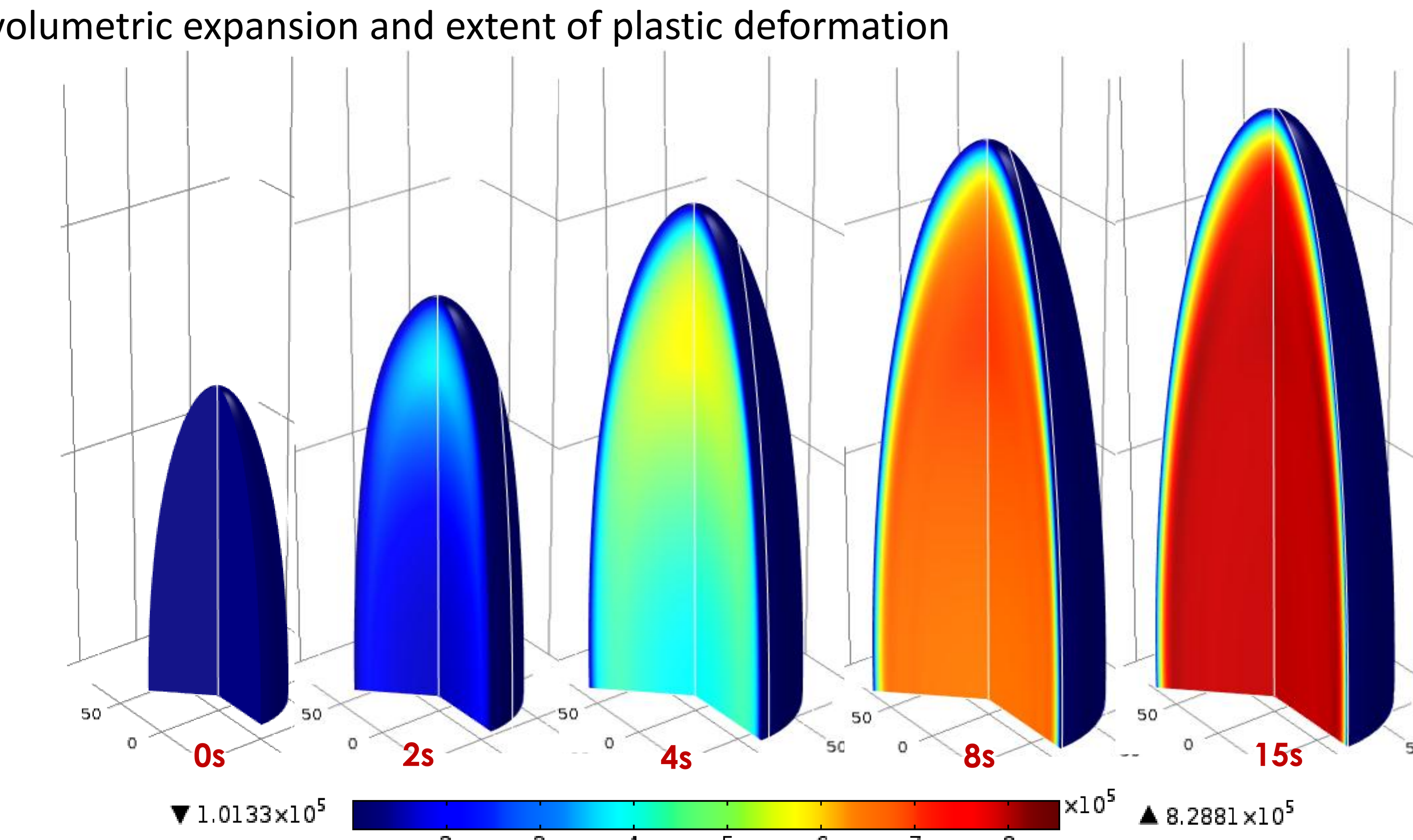
- Temperature, concentration, gas pressure and velocity

Solid Mechanics

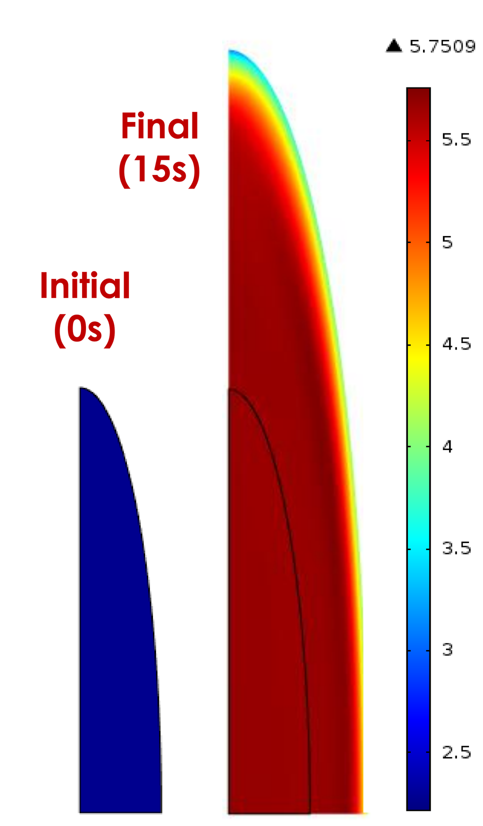
- 2nd PK Stress, Strains (Elastic and Plastic)

Results

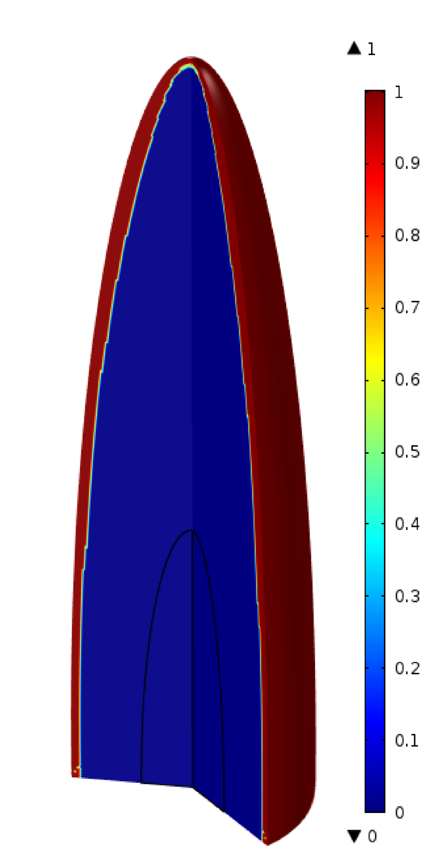
The coupled Transport and Solid Mechanical Model developed above was solved in COMSOL Multiphysics 4.3a to obtain the amount of gas pressure due to evaporation, volumetric expansion and extent of plastic deformation



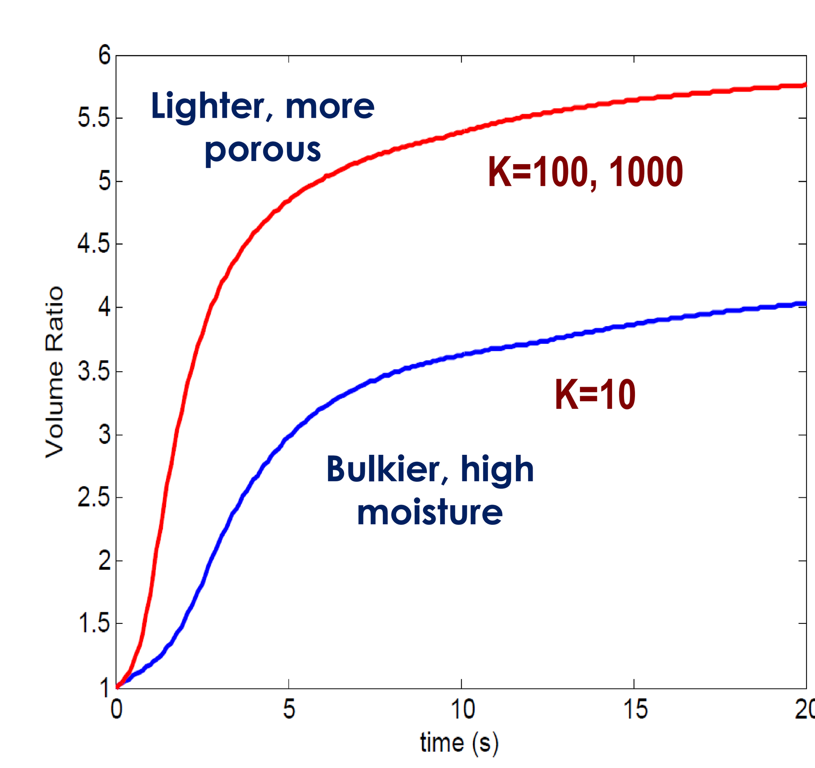
Simulated gas pressure generation with puffing time



Ratio of Initial Volume to Final Volume is around 6



Extent of Plastic Deformation of the kernel at the end of puffing



Evolution of Volume Ratio with time at different evaporation rates (K)

Summary and Conclusions

- A fundamentals-based model that predicts the volume expansion of rice kernels during hot air puffing is presented
- Rate of **evaporation** plays a significant role during puffing. **Volumetric expansion** is an important **quality attribute** associated with puffed products and it is sensitive to evaporation rate
- Rapid **evaporation** is required to generate **intense gas pressure** within the material in order to achieve a puffed product with high expansion ratio

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