Nusselt, Rayleigh, Grashof, And Prandtl: Direct Calculation of A User-defined Convective Heat Flux

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Abstract: When an electronic device is worn for extended periods, possibly in direct contact with human skin, heat must be safely transferred away from the device, without exceeding standards and regulatory temperature limits on the skin and on the exposed surfaces of the device. Heat transport is dominated by convective heat transfer to the surrounding air (possibly trapped air under clothing), and by blood flow cooling. This manuscript discusses each of these two mechanisms, including how to calculate and directly (non-iteratively) apply a "User defined" convective heat transfer coefficient, allowing more flexibility than the built-in external and internal convection options in COMSOL.

Keywords: heat transfer, worn device

1. Introduction

When an object such as an electronic device is in direct contact with human skin, heat is transported between the object and the skin. If the electronic device is hotter than the skin, heat is transferred to the skin until the device has cooled and the skin has heated sufficiently to be at the same temperature as the device. If the skin temperature in this steady-state condition is excessive, damage to the skin may occur. Studies using mice, pigs, guinea-pigs, and human volunteers in the 1930s-40s indicate that the threshold for damage is 44°C [1-5]. The threshold of irreversible damage may be higher, but less than 50°C, the temperature stated in ref. [4]. If the electronic device is very hot, damage can also occur before the steady state has been reached. For example, at 48°C, damage occurs after about 15 minutes [5]. In the temperature range from 44°C to 51°C, the time until damage occurs is roughly halved with every 1°C increase in temperature [5].

When designing an electronic device that will or potentially could contact human skin, the resulting heat increase of the skin must be considered to avoid thermal damage. In this manuscript we will use a finite element method computer simulation to establish a heat budget for an electronic device. The heat budget will satisfy our condition that the steady-state skin temperature must not reach the damage threshold of 44°C. (Transient temperatures and damage integrals are also calculated but not reported here.)

The established heat budget will also ensure that our electronic device obeys regulatory limits. Note that all temperatures in the previous paragraphs refer to the *skin* temperature. The International Electrotechnical Commission (IEC) has used findings of this type to set a standard on temperature limits on *objects* touching human skin. This standard is found in IEC 60601-1 [6]. The precise limits vary depending on factors such as duration of the touch and material properties of the object. In this manuscript we are interested in long-term contact. IEC 60601-1 treats contact longer than 10 minutes as steady state, so that effects of material properties vanish, and the temperature limit on the object equals the maximum steady-state skin temperature that does not cause damage, 43°C.

2. Governing equations and boundary conditions

We wish to set up a finite element method computer simulation that accurately predicts the human skin temperature where the skin is contacting a heat generating device. To do so we must model heat transport in the skin, in other tissue types near the skin, in the device, and in the surrounding area. The computational domain must be somewhat limited in scope to ensure a reasonably fast computation. This requires us to specify suitable boundary conditions on all domain boundaries.

Heat transport in the human body can be described by Pennes' equation [7], which is simply the regular equation for heat conduction with an added source term

$$Q_b = \rho_b C_{p,b} \omega (T_b - T)$$

that represents heat added or removed by blood flow. Subscript b here refers to blood properties, i.e., ρ_b is the blood density, $C_{p,b}$ is the blood heat capacity at constant pressure, and T_b is the blood temperature, assumed constant. The perfusion ω has units of inverse time (volume of blood per volume of tissue per unit time).

We will not model the entire human body, but rather a small section of it. The section consists of layers of different tissue types as shown in Fig. 1: skin, fat, muscle, and viscera. We specify the thickness of each layer and its thermal properties as indicated in Table 1. The total thickness of all layers equals half the width of a human body. Due to metabolism, each tissue type is also a heat source, generating a certain amount of heat per unit volume. The metabolism of the viscera is calculated so that the total metabolism of the modeled body section is in volumetric proportion to the metabolism of the entire human body. Blood is not in itself a significant metabolic heat source, but note that when the blood temperature T_b is larger than the local temperature T, the blood effectively *acts* as a heat source, and vice versa, when the blood temperature is smaller than the local temperature the blood acts as a heat sink.

The electronic device consists of various electronic components, including circuit boards and batteries, an enclosure, and small pockets of trapped air. Heat transport is dominated by conduction through metal parts, such as the circuit boards, wires, batteries, and the parts of the enclosure consisting of metal. Convection in the trapped air pockets is insignificant. Consequently we use a heat conduction equation to model the electronic device. We use several heat sources internal to the device, including batteries and individual electronic components.

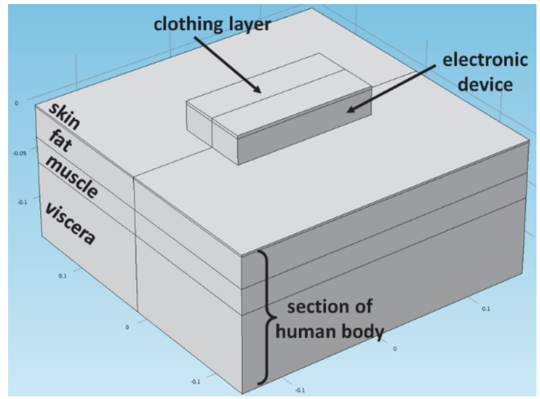

Figure 1. The computational domain consists of a section of the human body divided into four layers (skin, fat, muscle, and viscera), an electronic device, and a clothing layer (only partially shown).

Next we turn to the boundary conditions, beginning with those highlighted in blue in Fig. 2. Because the total thickness of the section equals half the width of a human body, the "interior" boundary condition is a symmetry boundary over which no heat flux takes place. Thus we can freely model this either with a symmetry boundary condition, a thermal insulation boundary condition, or a constant temperature equal to the body core temperature. Furthermore, we assume that the section of the human body is thermally independent, i.e., responsible for its own thermal regulation, and set thermal insulation boundary conditions at the other boundaries (we confirm the correctness of this assumption by checking that the effect of the electronic device does not extend out to the body section boundaries) and a heat flux boundary condition at the skin. The heat flux is the result of several complicated physical processes; the body regulates its temperature through various biological functions, including sweating and dilation or constriction of blood vessels. Fortunately, there is no need to model

Table 1. Tissue dimensions, material properties, and metabolic properties. Values from ref. [8] except where noted. The viscera value for perfusion was calculated as the average of the skin, fat, and muscle values. The viscera values for thermal conductivity, density, and heat capacity are organ values from ref. [8].

	thickness [mm]	metabolic rate [W/m^3]	perfusion ω [s^{-1}]	thermal conductivity [W/m·K]	density [g/cm^3]	heat capacity [J/kg·K]
skin	2	1300	0.0018	0.47	1.085	3680
fat	30	250	0.00043	0.16	0.85	2300
muscle	25	500	0.0005	0.42	1.085	3768
viscera	*net* [9]	*net* [10]	*average*	0.53	1.0	3697
blood	n/a	Q_b	n/a	n/a	1.0 [11]	4200 [11]

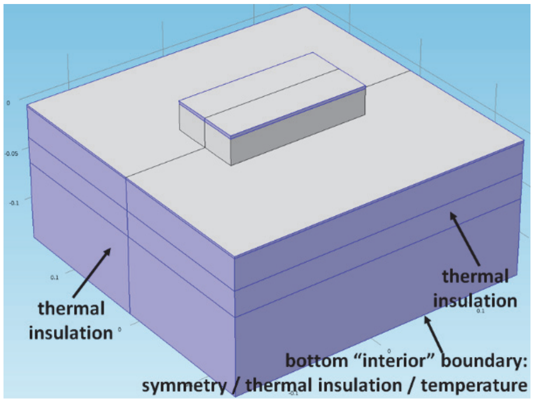

Figure 2. Thermal insulation boundary conditions are used to separate the section of the human body that is included in the computational domain from the rest of the body.

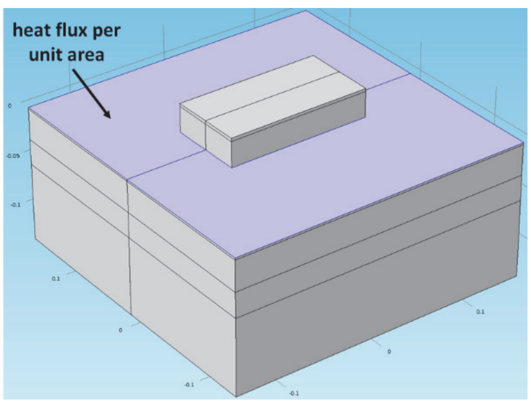

Figure 3. A heat flux per unit area is used as a boundary condition on the skin. The heat flux corresponds to that necessary for the body to regulate its temperature.

these details; without the electronic device present, the body regulates the heat flux so it precisely equals the total metabolism inside the body section. We can thus calculate a heat flux per unit area, which we then, when the electronic device is present, apply to the part of the skin not touching the electronic device, see Fig. 3.

The boundary conditions on the electronic device can vary depending on assumptions on the surrounding area, including the presence of clothes. Different sides of the electronic device have different boundary conditions, depending on the type of clothing (e.g., loose or tight clothes) and, for free convection, the orientation of the device relative to gravity. For this manuscript we have chosen, as an illustrative example, a situation where the electronic device is covered by fairly tight clothes that are stretched across the outer surface of the device. Thus, the outer surface

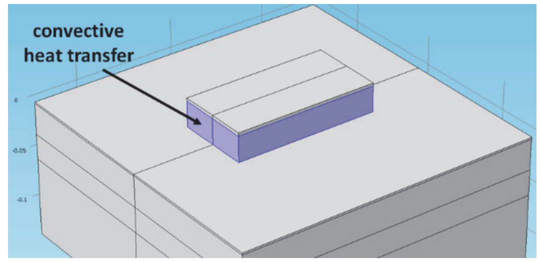

Figure 4. Convective heat transfer occurs from the electronic device to air trapped between the body and the clothing. Convective heat transfer boundary conditions are used on these surfaces.

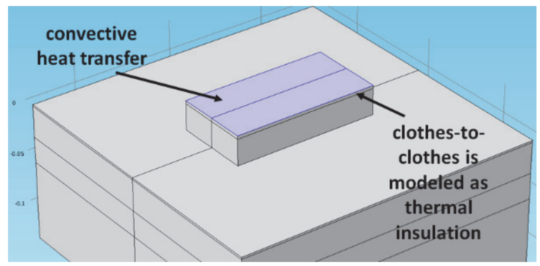

Figure 5. Another convective heat transfer boundary condition is used at the outer boundary of the clothing layer. Thermal insulation is used on the sliver surfaces where the modeled clothing layer touches the rest of the (unmodeled) clothing.

is touching a clothing layer, the inner surface is touching the skin, and the remaining sides are in contact with trapped air between the clothes and the body section. The outer and inner surfaces are not boundaries of our computational domain and do not need boundary conditions specified (although we eventually have to say something about the outer boundary condition of the *clothes*). As shown in Fig. 4, we will use a convective heat flux boundary condition on the sides, and assume that the trapped air temperature equals the skin temperature, $T_{skin} = 31.8°\mathrm{C}$.

The clothes are modeled by heat conduction using the material properties of air, and another convective heat flux boundary condition is applied on the outer surface of the clothes, as shown in Fig. 5, now flowing into an ambient temperature of 26°C. On the tiny sliver boundaries where our modeled clothing layer touches the rest of the clothing layer we apply a thermal insulation boundary condition; this is not entirely correct, but these surfaces are much too small to have an impact.

3. Use of COMSOL Multiphysics

The problem was set up in base COMSOL (without additional modules), as a 3D geometry and using the Heat Transfer in Solids physics. Heat source nodes were added for the different tissue types (skin, fat, muscle, and viscera), for the blood, and for various components in the electronic device.

For the blood heat source term we chose to manually type it in, simply typing in the Pennes' source term above, and using a perfusion defined under Global Definitions as a Piecewise Constant function (constant within each tissue type layer). A user who finds this complicated could instead use the Heat Transfer Module, where Bioheat Transfer with Pennes' equation is pre-defined. The total blood heat was calculated using an Integration Component Coupling, and was set to zero by adding a Global Equation under a Global ODEs and DAEs node and using the skin temperature as the state variable. (More about Global ODEs and DAEs for the beginner COMSOL user below.) We found that the skin temperature calculated this way came out to 31.7°C, a nearly perfect match to the tabulated 31.8°C value [10].

To calculate the temperatures correctly, it is essential that all convective heat transfer (from device to air trapped under the clothing, from clothing to ambient air) are handled properly. COMSOL has several choices of built-in heat transfer coefficients that work well as boundary conditions in many situations. However, in this example we desired greater control of the calculations and chose a Heat Flux node with a Convective heat flux and a "User defined" Heat transfer coefficient. Instead of specifying a fixed number for the heat transfer coefficient h_c, we calculated it from:

$$h_c = \frac{Nu \cdot k}{L}$$

$$\mathrm{Nu} = C \cdot \mathrm{Ra}^n$$

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr}$$

$$\mathrm{Gr} = \frac{gL^3}{\eta^2}\left(\frac{T_p}{T_a} - 1\right)$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}$$

where $k = k(T_a)$ is the heat conductivity of air, $\eta = \eta(T_a)$ is the kinetic viscosity of air, $\mu = \mu(T_a)$ is the dynamic viscosity of air, $C_p = C_p(T_a)$ is the heat capacity of air, and g is the acceleration of gravity. We calculate the coefficient from a series of four dimensionless numbers, the Nusselt, Rayleigh, Grashof, and Prandtl numbers. These were defined as variables in COMSOL. Note that because the calculated heat transfer coefficient is itself also a variable, it cannot be directly typed into the box for the heat transfer coefficient in the Heat Flux node (this would effectively cause a circular definition). Instead, we define an extra equation that ensures that the difference between the calculated coefficient h_c and the specified coefficient $h_{c,SpecifiedInNode}$ is zero. This is done by creating a Global ODEs and DAEs physics and specifying a Global Equation where

$$h_c - h_{c,SpecifiedInNode} = 0.$$

In reality, we used several sets of heat transfer coefficients, dimensionless numbers, etc., representing different sides of the electronic device and the clothing to ambient interface. Table 2 shows examples of values used. Note that one advantage of using "User defined" heat

Table 2. Parameters used for calculations of the Nusselt, Rayleight, Grashof, and Prandtl numbers, and the convective heat transfer coefficients. Note that the temperature T_p was calculated taking average temperatures over various sides of the device (or clothing) using Average Component Couplings in COMSOL.

	L	C	n	T_p	T_a	valid for
module sides	y_m	0.59	0.25	$\overline{T_{side}}$	T_{skin}	10^4<Ra<10^9
module top	$x_m/2$	0.54	0.25	$\overline{T_{top}}$	T_{skin}	10^4<Ra<10^7
module bottom	$x_m/2$	0.27	0.25	$\overline{T_{bottom}}$	T_{skin}	10^5<Ra<10^{11}
clothing-to-ambient	y_m	0.59	0.25	$\overline{T_{face}}$	$T_{ambient}$	10^4<Ra<10^9

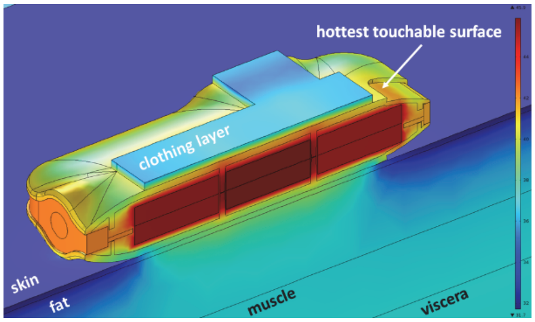

Figure 6. Simulation of an electronic device touching human skin, and with a clothing layer on top of the device. The cut-way view shows only portion of the human and the clothing, and cuts the device in half. Color represents temperature, with the deepest red corresponding to 45.9°C (in the interior of the device) and dark blue corresponding to 31.7°C.

transfer coefficients in this manner is that different design choices, including small scale surface textures that we would prefer to not have to simulate, can be represented by changes to the values in Table 2.

Finally it may be worthwhile reminding the reader that all uses of heat transfer coefficients are simplifications that entail some amount of inaccuracy, especially if the geometry is not simple. For full accuracy, the actual air flow has to be calculated. This is beyond the scope of this manuscript.

4. Results

Figure 6 shows the COMSOL simulation of a 141 mm by 83 mm by 25 mm electronic device oriented with its long axis horizontally, and sandwiched between a tightly stretched 3 mm clothing layer and a section of the human body. The ambient temperature was 26°C, the human was considered at rest (at 13% VO_2 max) with a core temperature of 36.7°C and a skin temperature of approximately 31.7°C. It was found in the simulation that the device could release 1.75 W of heat without any external surface of the device exceeding 43°C. The maximum skin temperature under these conditions would be 39.4°C. If the heat release is increased by another watt to 2.75 W, the skin would reach 42.2°C, but some portions of the device not touching the skin would then exceed 43°C, with one small patch reaching as high as 49.5°C. Of course, if a person were to touch this high temperature patch, its temperature would drop. To know whether this would cause damage, a weighted time integral of the temperature (a “damage integral”) would have to be calculated.

8. Conclusions

Through the use of finite element method numerical simulations, we were able to establish a heat budget for an electronic device that we are designing.

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