Numerical Simulation of the Thermal Response Test Within Comsol Multiphysics® Environment

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Abstract:
An estimation method, known as Thermal Resposne Test, of the soil thermal properties necessary to the design of a borehole geothermal energy storage system is discussed in relation to its application to ground having non–homegeneous composition. The governing equations of the conduction/convection heat transfer unsteady problem which describe the system behaviour have been solved within Comsol Multiphysics® environment.

Keywords: Geothermal borehole heat exchanger, Thermal Response Test.

1. Introduction

Borehole thermal energy storage systems represent a convenient tool for exploiting effectively the heat capacity of the soil [1,2]. Their use can bring significant benefits in terms of energy savings in space heating/cooling if compared to conventional systems. The correct design of the borehole energy storage system requires the knowledge of the soil thermal properties. For this aim, an estimation procedure, named the Thermal Response Test, which allows in-situ-determination of the equivalent or effective ground thermal conductivity and borehole thermal resistance, is often adopted, especially in vertically oriented loops. This approach is based on the comparison between the analytical solution of an unsteady heat conduction problem within the soil and the average fluid temperature experimentally acquired, directly on the borehole heat exchanger. This approach was first adopted by Morgensen [3], who considers the so called line source model in order to approximate the borehole storage system's behaviour. This simple model consists on the solution of the unsteady heat conduction problem in a semi-infinite isotropic homogeneous constant properties medium in which a line heat source is present. More complex models have been suggested, like the Cylinder Source Model [4] to take into account the finite dimension of the heat source. A more general estimation approach, based on the direct numerical solution of the partial differential equations governing the phenomenon [5,6], has been suggested, too.
The major weaknesses of the Thermal Response Test with regards to the design of underground thermal energy storages are due to the fact that it doesn't allow to determine the volumetric heat capacity of the system, and therefore its thermal diffusivity, which is a necessary information in order to correctly asses the performance of the heat exchanger. Moreover it assumes the soil as infinite homogeneous medium with constant properties, while very often its composition can significantly vary with increasing depth. Another important aspect which might impair the results obtained by means of the Thermal Response Test based on the line source model, is related to the boundary and initial conditions, which, in the theoretical model, are respectively assumed of constant temperature in the undisturbed region and of uniform temperature while they are often a more complex function of space and time in the real system. In the present work the finite element method, implemented within the Comsol Multiphysics® environment has been adopted to solve the partial differential equation governing the heat transfer problem in a tube-in-tube borehole energy storage system. The two-dimensional transient conduction heat transfer problem within the soil has been coupled to the one-dimensional convective problem within the carrier fluid, by means of the *weak boundary condition*, according to the scheme outline in [7]. The constant heat rate to the fluid is approximated by means of a periodic temperature boundary condition. The comparison of the numerical results with the analytical solution of the line source model problem, enables to discuss the capability of the Thermal Response Test with regards to the characterization of borehole energy storage

systems also in conditions in which the composition, and therefore the thermal properties, of the soil are non uniform.

1.1 The model of the borehole heat exchanger

The scheme of the geothermal heat exchanger and of the coupled energy storage system considered in the present analysis, is schematically shown in Fig.1. It consists of a tube-in-tube pipe with downward flow in the inner section, having radius r_1 = 0.0165m, and upward flow in the annular section, having external radius r_2 = 0.05m. This configuration is intended to simulate the U-tube configuration, generally found in these type of heat exchangers. For the sake of simplicity, the wall thickness of the pipe was disregarded .
The grout fills the space between r_2 and the radius of geothermal heat exchanger, r_b = 0.075 m. The system has an axial symmetry and it is considered practically unlimited in the radial direction ($R \rightarrow \infty$), while in axial direction it is limited by two adiabatic surfaces placed at z = 0 (soil surface) and z = 100 (deep end of the heat exchanger).

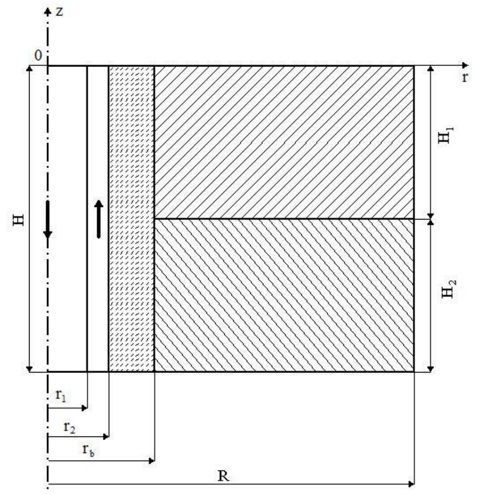

Figure 1. Geometry of the geothermal borehole system considered.

1.2 The Line Source Model

By assuming that the soil is a uniform and isotropic medium and also that the temperature difference between the inlet and outlet section of the heat exchanger remains constant over time, the thermal unsteady problem in the system can be approximately solved by considering the geothermal heat exchanger as a linear heat source, which suddenly releases a finite, uniform and constant quantity of energy in a homogeneous medium unlimited in the radial direction and having uniform initial temperature.
For this model, known as the line source model, the analytical solution [8] is:

$$T(r,t) = T_0 + Q/(4\pi\lambda H) E_1[r^2/(4\alpha t)] \qquad (1)$$

where E_1 is the integral exponential function. For sufficiently large values of the parameter $4\alpha t/r^2$, Equation (1) can be approximated by:

$$T(r,t) \cong T_0 + Q/(4\pi\lambda H)[\ln(4\alpha t/r^2) - \gamma] \qquad (2)$$

The temperature at the interface between the heat exchanger and soil ($r = r_b$) can be obtained as follows:

$$T(r_b,t) \cong T_0 + Q/(4\pi\lambda H)[\ln(4\alpha t/r_b^2) - \gamma] \qquad (3)$$

By defining the thermal resistance per unit length between the working fluid and the cylindrical surface at $r = r_b$, as follows

$$R_b = [T_f(t) - T(r_b,t)]H/Q \qquad (4)$$

where T_f is the mean fluid temperature it can be concluded that:

$$T_f(t) \cong T_0 + Q/(4\pi\lambda H)[\ln(4\alpha t/r_b^2) - \gamma] + \\ + R_b Q/H \qquad (5)$$

Equation (5) is the mathematical model to which the Thermal Response Test usually refers in order to estimate the thermal equivalent conductivity of the soil and the borehole thermal resistance, too.
It should be noted that use of equation 5, implicitly assumes the following simplifications:
-the thermal properties of the heat exchanger and soil are the same;

-the pipe in which the working fluid flows is placed on the symmetry axis of the system and it has a negligible diameter;
-the fluid temperature doesn't change along the axial direction.
Accordingly, the model is expected to give a poor approximation of the real system behaviour in the early regime, when the capacitive effects of the heat exchanger are particularly relevant.
Equation (5) can be conveniently rewritten placed in the form:

$$T_f(t) \cong m + k\ln(t) \tag{6}$$

Where

$$k = Q/(4\pi\lambda H) \tag{7}$$

And

$$m = T_0 + Q/(4\pi\lambda H)[\ln(4\alpha/r_b^2) - \gamma] + \\ + R_b Q/H \tag{8}$$

The estimation procedure is based on the comparison, under a least square approach, between the temperature of working fluid, experimentally acquired and evaluated as the arithmetic mean between the inlet and outlet fluid temperature, and equation (6). From the knowledge of the coefficient *k*, the equivalent thermal conductivity of the soil can be easily derived. Moreover, from the estimated value of *m*, the borehole thermal resistance R_b is also recovered, by assuming that the soil thermal diffusivity is known. In particular, this last assumption is based on the hypothesis that the thermal capacity per unit volume of the ground is known. In fact, in the practical application of the Thermal Response Test, for the soil volumetric thermal capacity it is generally adopted a value which is assumed typical of most type of soils. However, it should be noted that the variation of the soil thermal capacity per unit volume may be of the same order of magnitude of the variation shown by the thermal conductivity. For example, for some materials frequently present in the soil (clay, grit, sand, limestone), the ASHRAE Handbook [1] reports thermal conductivity values ranging between 1.4 and 5.2 W/m°C, also depending on the water content, while the corresponding volumetric thermal capacity values range between 1.6 and 3.6 °C MJ/m^3.

Regarding this, the results reported in [9] partially confirm the propriety of this approach in relation to the design of geothermal borehole energy storage systems. In particular they show that the thermal capacity per unit volume, and therefore the thermal diffusivity, has a minor effect on the estimation of the soil thermal conductivity and of the heat flux exchanged per unit length only under the hypothesis that the ground energy storage surrounding the borehole extends indefinitely in the radial direction. In situation when the medium in which the borehole is immersed cannot be considered ~~a~~ semi-infinite, the thermal diffusivity has instead a significant effect on the heat flux exchanged by the heat exchanger.
Another aspect which can limit the predictive capability of the Thermal Response Test is related to the effect of non-homogeneity of the soil. The present analysis is in particular focused on the validation of the estimation approach based on the Thermal Response Test to situation in which the soil is not homogeneous in composition and therefore its thermal properties may vary with depth. To this aim, the governing partial differential equation, have been implemented and solved within Comsol Multiphysics® environment by considering different schematic cases, corresponding to soil media non-homogeneous with respect to both thermal conductivity and/or volumetric heat capacity.

2. Governing equations and solution method within Comsol Multiphysics®

Transient heat transfer conduction is governed by the Fourier equation, which, under the assumption of homogeneous medium, is:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \mathrm{div}\left(\overrightarrow{\mathrm{grad}\, T}\right) \tag{9}$$

For the thermal problem schematized in fig. 1, eq. (9) has been solved in each homogeneous domain with the initial condition:

$$T(r,0) = T_0 \tag{10}$$

At r = r_2 the thermal boundary condition is described by:

$$-\lambda \frac{\partial T_F}{\partial r}\bigg|_{r=r_2} = h_o\left[T_o(z,t) - T_F(r_2,t)\right] \qquad (11)$$

being h_o the convective heat transfer coefficient, derived from the Dittus-Boelter correlation, associated to the working fluid, flowing in the annular section with temperature T_o.
By assuming that the convection problem both in the tube and in the annular section of the heat exchanger is one-dimensional, and by modelling the thermal coupling between the two counter-current stream through the thermal conductance per unit length U, the energy equation for the tube side fluid flow:

$$A_i \rho_f c_{pf}\left(\frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial z}\right) = U\left[T_o(z,t) - T_i(z,t)\right] \qquad (12)$$

with the initial condition:

$$T_i(z,0) = T_{i,0} \qquad (13)$$

being T_i the tube-side fluid temperature and u_i the fluid mean velocity in the axial direction.
The corresponding equation for the annular section is:

$$\begin{aligned} &A_o \rho_f c_{pf}\left(\frac{\partial T_o}{\partial t} + u_o \frac{\partial T_o}{\partial z}\right) = \\ &U\left[T_i(z,t) - T_o(z,t)\right] + h_o p\left[T_F(r,t) - T_o(z,t)\right] \end{aligned} \qquad (14)$$

with the initial condition:

$$T_o(z,0) = T_{o,0} \qquad (15)$$

being T_o the annular-side fluid temperature and u_o the fluid mean velocity in the axial direction.
The condition of constant power supplied to the working fluid is implemented by the condition:

$$T_i(0,t) = T_o(0,t) + \Delta T \qquad (16)$$

with ΔT constant over the whole temporal domain.
The U-tube configuration, commonly found in borehole geothermal heat exchanger, has been here simulated by imposing that the temperature of the tube-side downward flow equals the temperature of the upward annular-side flow at the end of the heat transfer section , as follows:

$$T_o(H,t) = T_i(H,t) \qquad (17)$$

The continuity condition of both temperature and heat flux at the interface between solid domains of different thermal proprieties, completes the statement of the problem.
The above equations have been solved by means of the finite element method implemented within the Comsol Multiphysics® environment. A 2-D model, discretized by means of rectangular axisymmetric elements has been adopted for solving the unsteady conduction heat transfer problem in the solid domains, while a 1-D model, implemented by means of the *weak form formulation* with elements distributed along the axial coordinate, has been adopted for solving the energy equation in the fluid domain.

3. Results

The effect of the thermal properties variability due to the soil non-homogeneity with depth has been considered, according to the data-scheme reported in table 1.

Type of soil	Composition	Thermal Conductivity (W/mK)	Thermal heat capacity per unit volume (MJ/m^{3}K)
A	H_1 = 0.2 H	2	2
	H_2 = 0.8 H	4	2
B	H_1 = 0.2 H	2	2
	H_2 = 0.8 H	4	3
C	H_1 = 0.5 H	1	2
	H_2 = 0.5 H	1	3
D	H_1 = 0.5 H	1	2.5
	H_2 = 0.5 H	1	2.5

Table 1. Soil types considered.

Regarding the thermal properties of the filling material, the typical values of sand, having thermal conductivity and volumetric heat capacity equal respectively to 2.1 W/m°C and 2.4 MJ/m^3°C, have been assumed. Water, with a mass flow rate of 0.5 kg/s, has been adopted as working fluid.

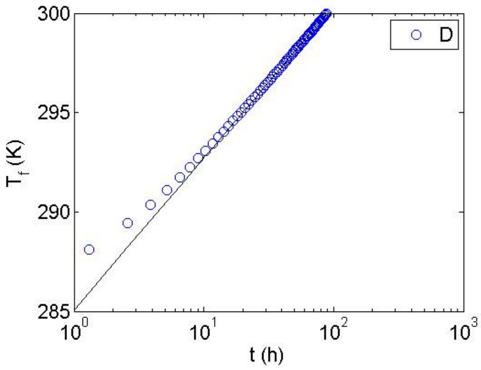

Figure 2. Average fluid temperature versus time for soil type D.

The fluid temperature difference, ΔT, between the inlet and outlet sections has been imposed equal to 2°C, that is an average heat power per unit length equal to 41.86 W/m has been considered supplied constantly to the soil. The thermal conductance per unit length, **U**, accounting for the thermal coupling between the tube-side and annular side flows, has been assumed equal to 14.14 W/m°C.

The average fluid temperature, reported in figure 2 for case D, shows that after a sufficiently long time, a linear trend with time in a semi-logarithmic scale, is reached, as predicted by equation (6). In the same figure the best-fit line according to the Line Source Model and obtained by considering a fitting period of 24 h, staring from t=100h, is reported too.

This behaviour has to be considered representative of all other cases considered in the present analysis and listed in table 1, regarding both homogeneous and non-homogeneous soils.

The temperature distribution along the axial coordinate at the borehole-soil interface ($r = r_b$) is reported in figure 3 for cases C and D, describing respectively a non-homogenous and a homogeneous soil~~s~~ with equal mean volumetric heat capacity for a given thermal conductivity value.

The curve obtained for the soil type C shows a discontinuity in the slope at the interface between the two layers having different volumetric thermal heat capacity.

The thermal conductivity, estimated according to equation (6) by considering a fitting period of 24h is reported in figures 4-7 versus the initial fitting time, for each soil type considered in the present analysis.

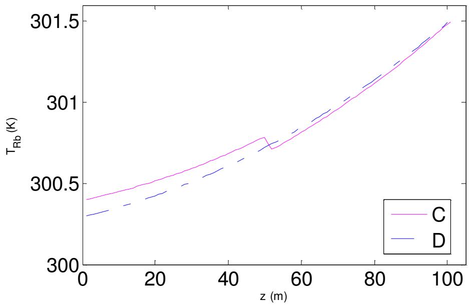

Figure 3. Temperature axial distribution at the borehole-soil interface for cases C and D.

In the case of non-homogeneous soil thermal conductivity (case A) and of both non-homogeneous thermal conductivity and volumetric heat capacity (case B), the estimated effective value of λ approaches, as expected, the value obtained by performing a mean, weighted according to the composition, of the values characterizing the single soil layer (value equal for both cases A and B to 3.6 W/mK).

In particular for cases A and B the effective thermal conductivity reaches this value, within a 3% approximation, respectively after 30 and 10 h.

When considering the volumetric heat capacity non-homogeneity only (case C), the thermal conductivity is recovered exactly already after 10h after the beginning of the transient.

Regarding the estimation of the borehole thermal resistance performed by means of the Thermal Response Test as outlined in the previous paragraph, it should be noted that this quantity is properly defined in a steady state regime. Therefore, equation (4) can be used to correctly evaluate the geothermal heat exchanger thermal resistance only when the related quantities remain constant over time and, consequently, when the system is not affected by capacitive effects. Moreover equation (4) assumes the temperature field to be one-dimensional, while in composite soils two-dimensional effects might become significant.

The borehole thermal resistance estimated according to the fitting procedure described by equation (8) is reported in figures (8) and (9) for cases C and D, describing respectively a non-homogenous and homogeneous soils with equal mean volumetric heat capacity for a given

thermal conductivity value. For the case of non-homogeneous soil, described by case C, the weighted mean volumetric heat capacity has been adopted in deriving the borehole thermal resistance value by means of the fitting procedure (equation (8)).
Again this approach provides, as expected, a good approximation since, when the capacitive effects becomes negligible (late regime), the two asymptotic values assumed by the borehole thermal resistance of cases C and D differ by less than 3%.

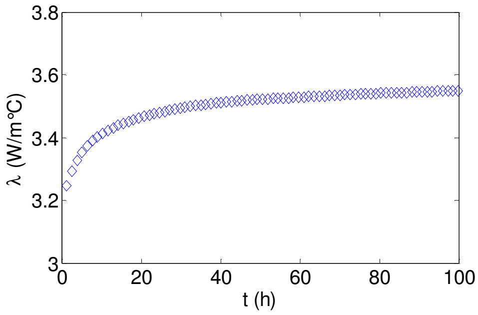

Figure 4. Estimated thermal conductivity versus initial fitting time for soil type A.

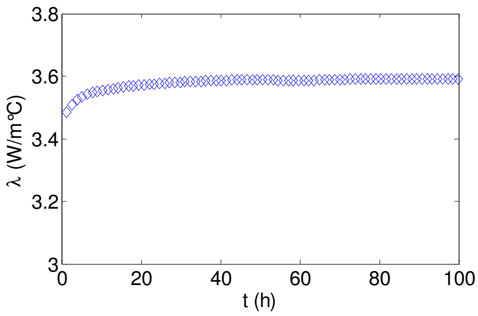

Figure 5. Estimated thermal conductivity versus initial fitting time for soil type B.

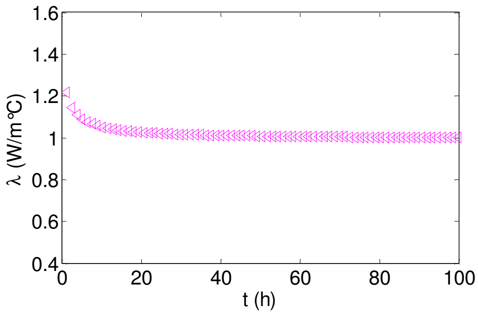

Figure 6. Estimated thermal conductivity versus initial fitting time for soil type C.

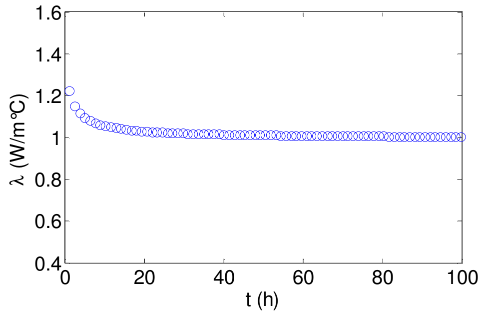

Figure 7. Estimated thermal conductivity versus initial fitting time for soil type D.

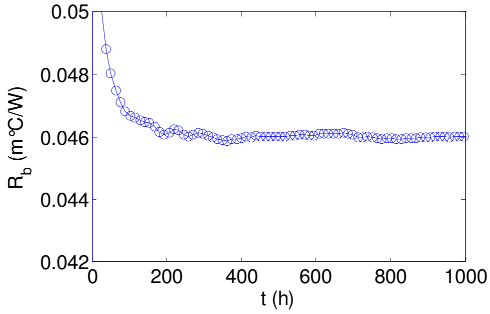

Figure 8. Estimated borehole thermal resistance versus initial fitting time for soil type C.

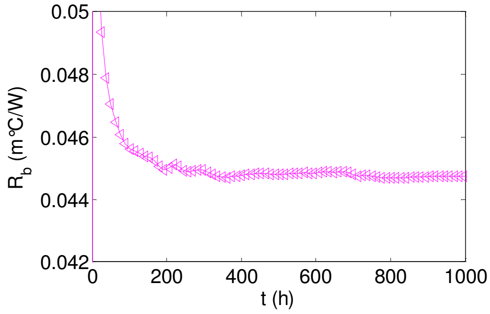

Figure 8. Estimated borehole thermal resistance versus initial fitting time for soil type D.

4. Conclusions

The governing equation of the conduction/convection heat transfer phenomena describing the behavior of a borehole geothermal heat exchanger have been solved within Comsol Multiphysics® environment. In particular the analysis has been focused on the discussion of the application of an estimation procedure, named Thermal Response Test, generally adopted to predict the soil thermal conductivity and the borehole thermal resistance, to non-homogeneous soils. The analysis confirms that, under certain hypothesis, the estimated properties resulting from the Thermal Response Test can be interpreted as effective values which approach the mean, weighted according to the soil composition, of the values characterizing the single ground layers.

5. Nomenclature

c_p	Specific heat at constant pressure	J/kg°C
H	Heat exchanger length	m
Q	Heat flux	W
r	Radial coordinate	m
R	Radius of the energy storage	m
R_b	Thermal resistance	°Cm/W
t	Time	s
T	Temperature	°C, K
u	Fluid velocity	m/s
z	Axial coordinate	m
α	Thermal diffusivity	m^2/s
γ	Eulero costant ≅ 0.57721	
λ	Thermal conductivity	W/m°C
ρ	Density	kg/m^3

Subscripts

b	Geothermal heat exchanger
f	Working fluid
F	Filling material
0	Initial value

6. References

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